Noncooperative Game Theory for Autonomous Racing

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Game-theoretical Motion Planning

ICRA 2021 - Tutorial

ETH zürich

CVL Computer Vision Lab

Automatic Control Laboratory IfA

Motivation

Autonomous driving

- Active research area since the 1980s
  - Research done in industry and academia
  - Waymo/Google: > 20 mio miles
- Take safety critical decisions in an uncertain environment

Autonomous racing

- Drive as fast as possible around a track
  - Miniature race car set-up using RC cars
  - Formula Student Driverless
  - Roborace
  - IndyAutonomous
- **Structured** but **competitive** environment

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Autonomous Racing Challenges

Driving at the handling limit

- If we do not drive at the limit we drive too slow
- Motion planning for a highly nonlinear system

Staying safe inside the track

- If we crash we lose!
- Infinite horizon constraint satisfaction

Interacting with other race cars

- The art of overtaking and interacting with other cars
- Decision making in a highly dynamical non-cooperative environment

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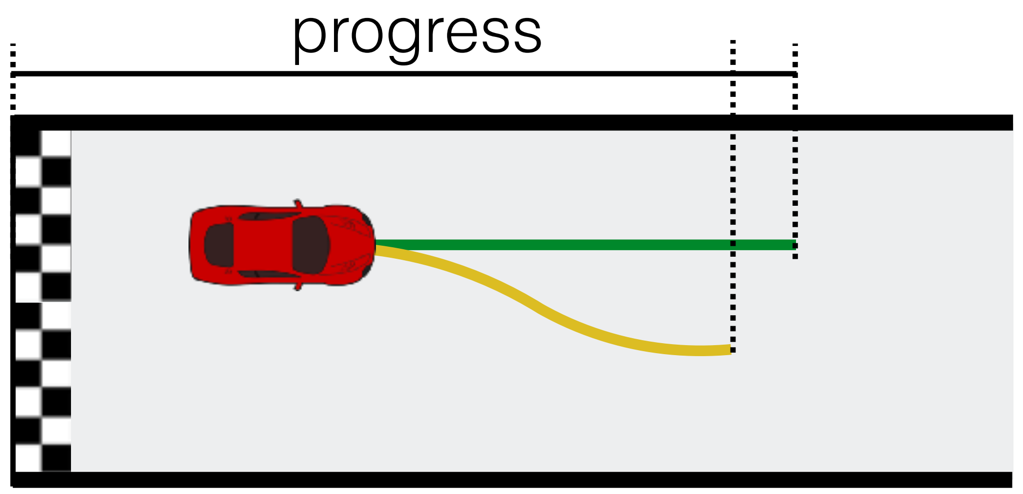
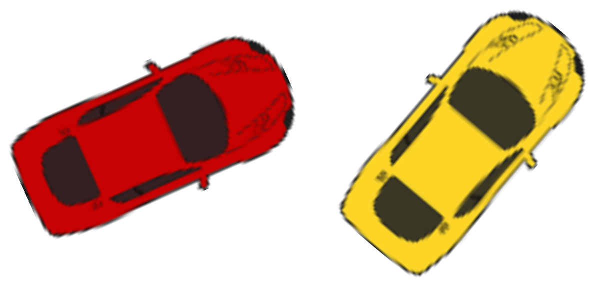
Racing Ingredients

Finish first

- Approximated by maximizing progress
- Generates racing trajectories



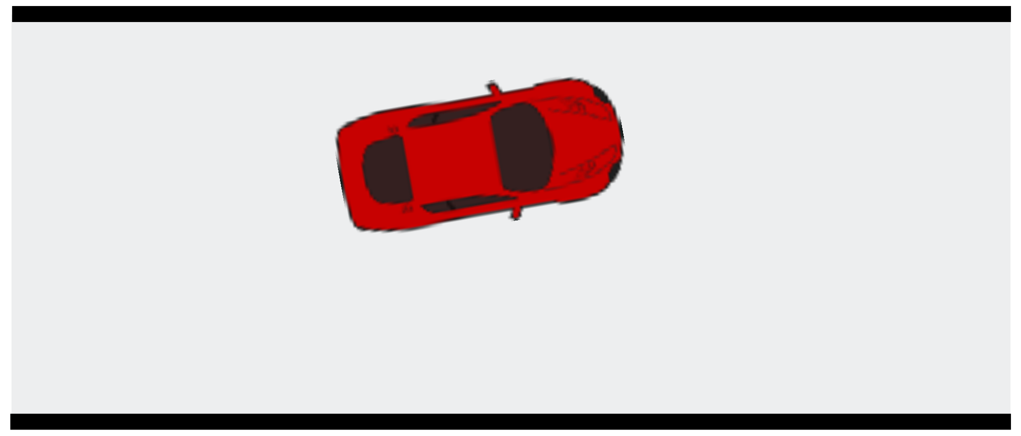
Do not collide with other cars



good

bad

Stay inside the track



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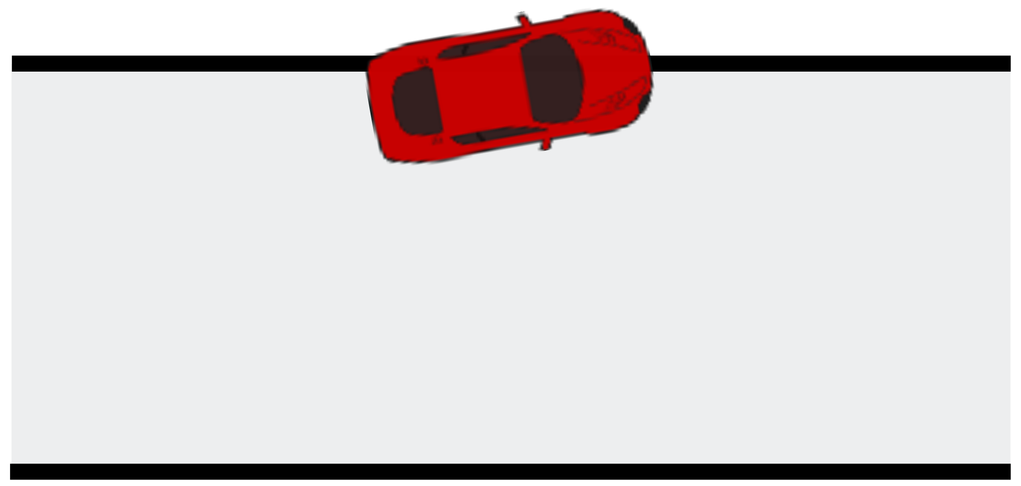

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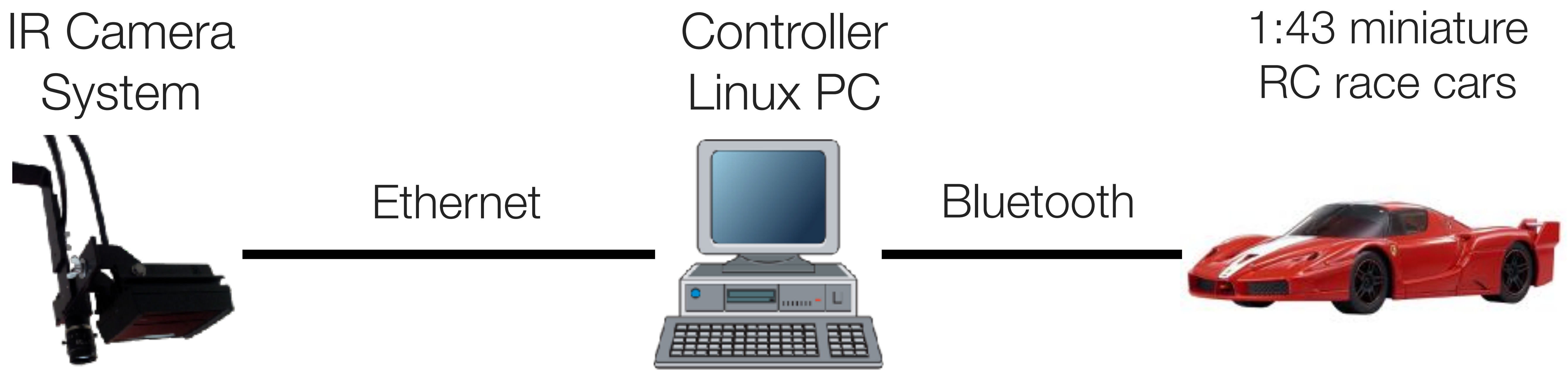
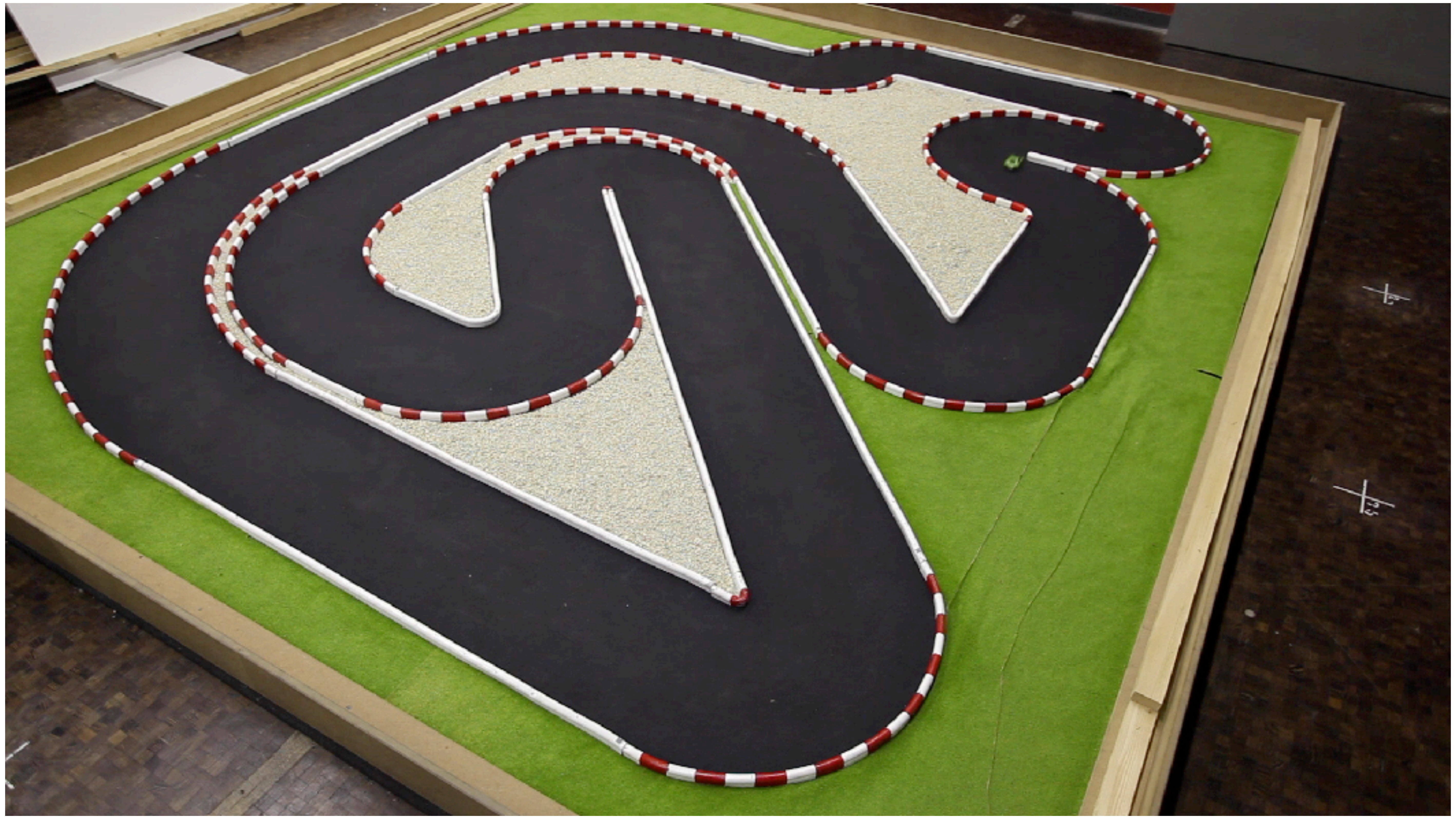
Experimental Set-Up





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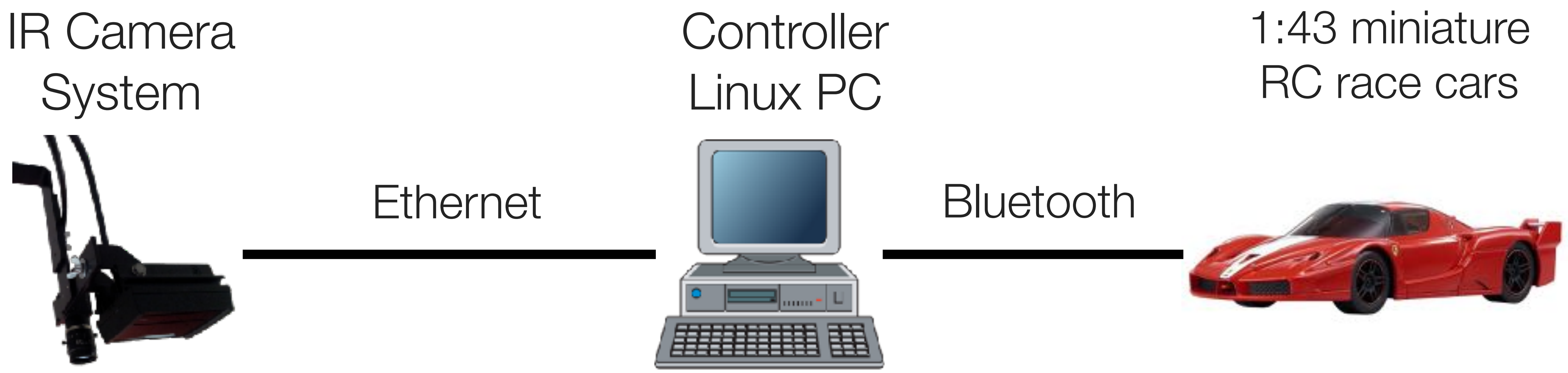
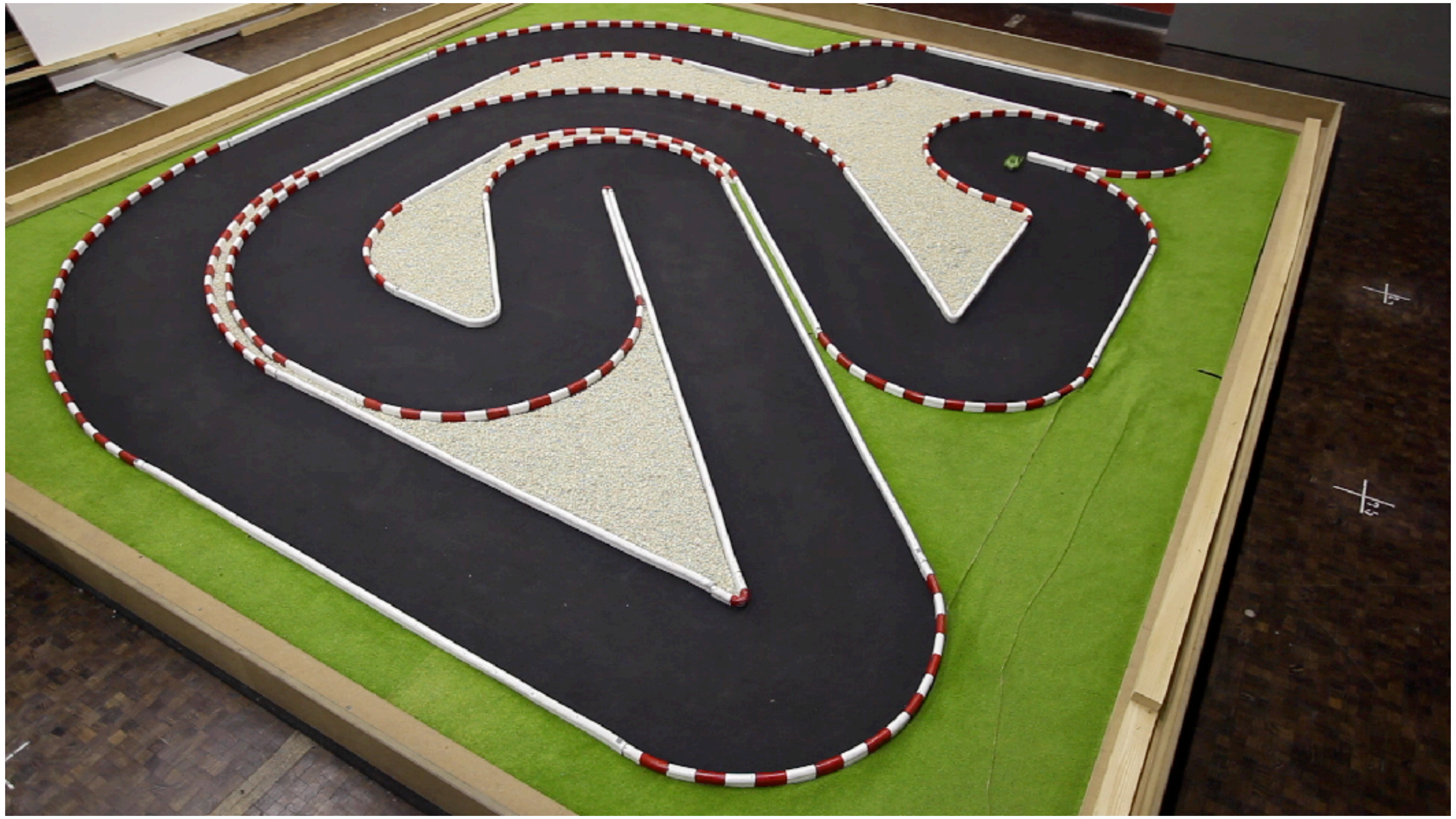
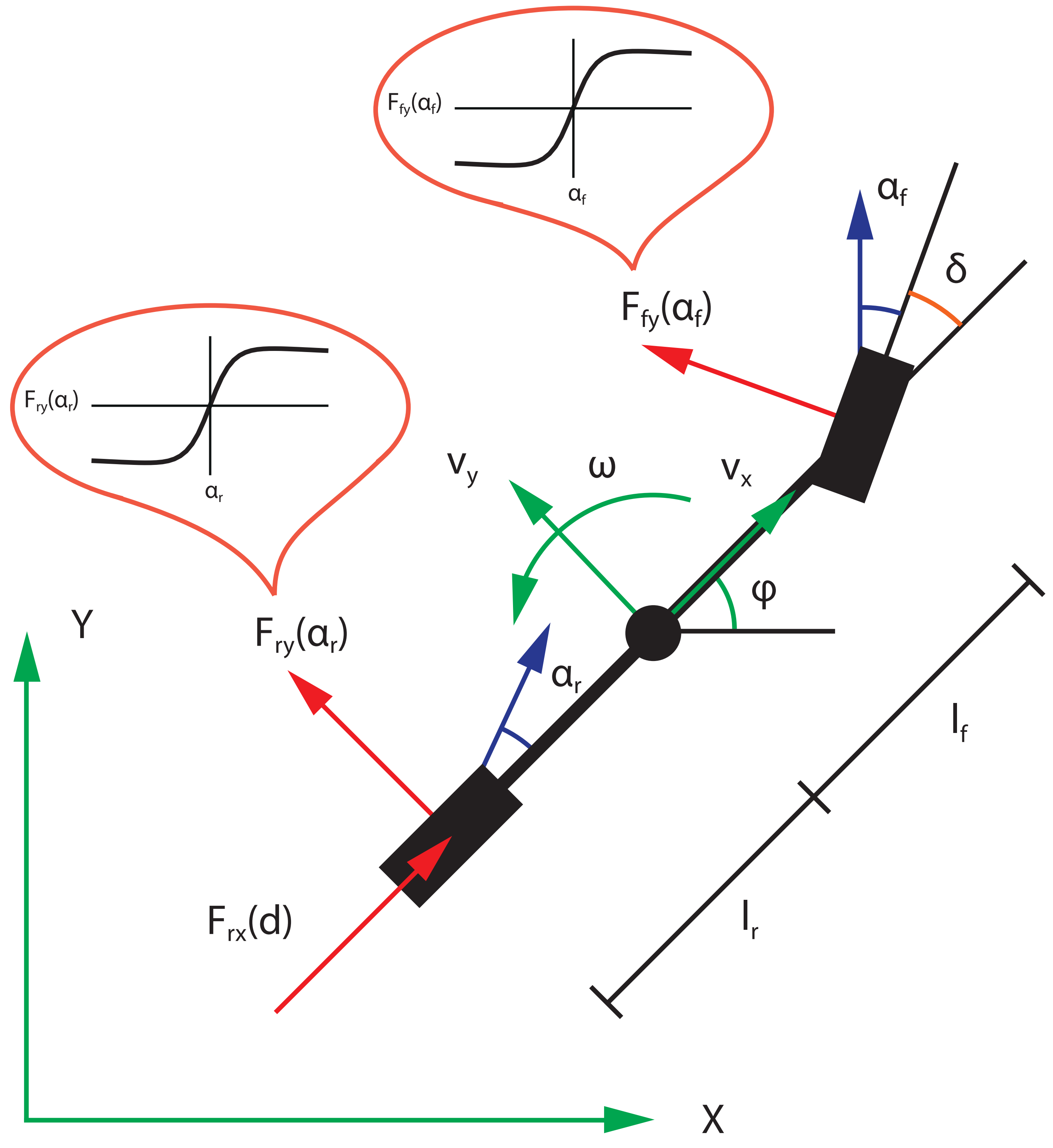




Car Model

- Bicycle model, with nonlinear lateral tire forces (Pacejka)



$$\dot{X} = v_x \cos(\varphi) - v_y \sin(\varphi)$$
$$\dot{Y} = v_x \sin(\varphi) + v_y \cos(\varphi)$$
$$\dot{\varphi} = \omega$$
$$\dot{v}_x = \frac{1}{m}(F_{r,x} - F_{f,y} \sin\delta + m v_y \omega)$$
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- Highly nonlinear 6 dimensional system

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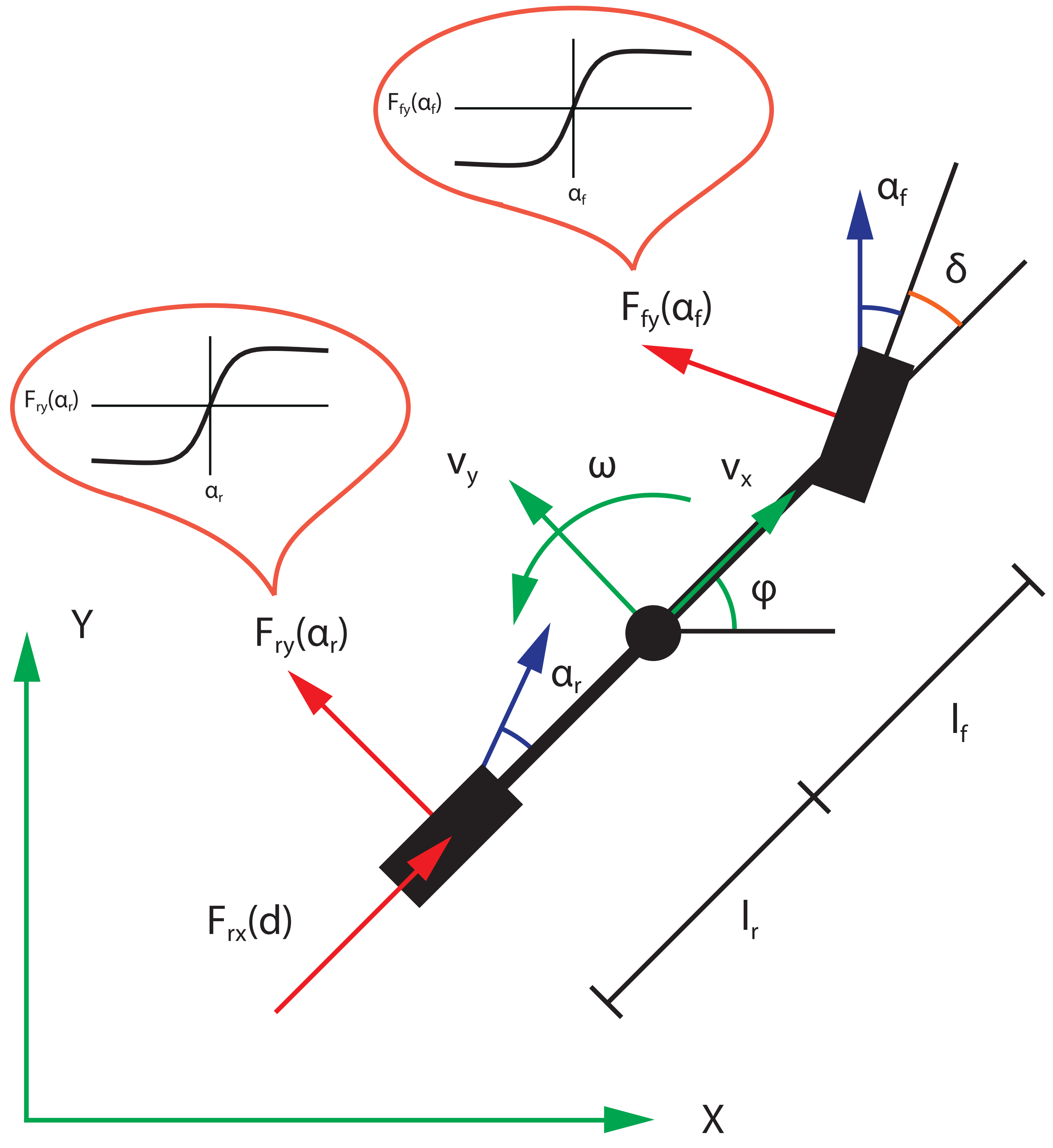

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- Separation in slow and fast dynamics

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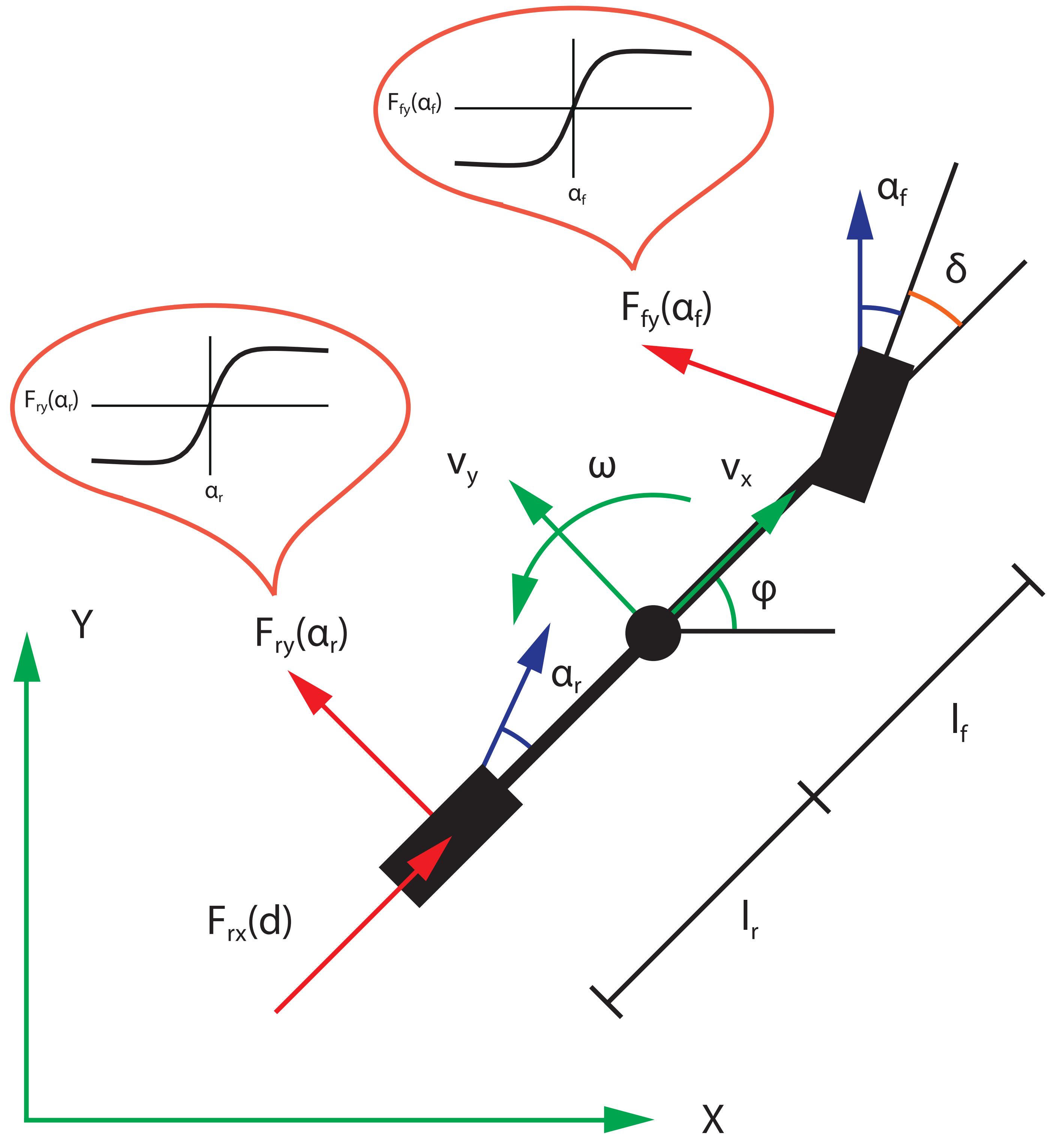

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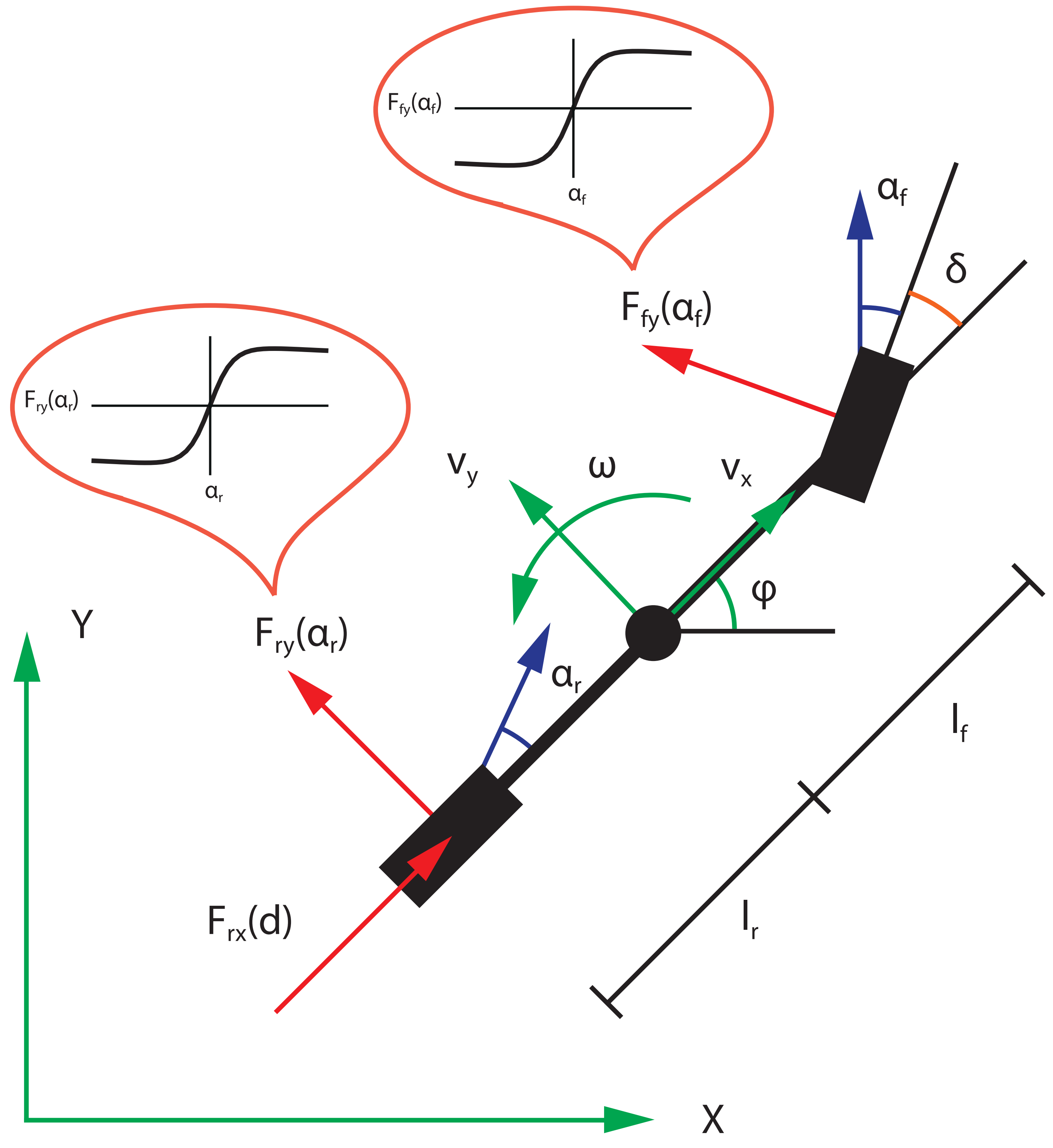

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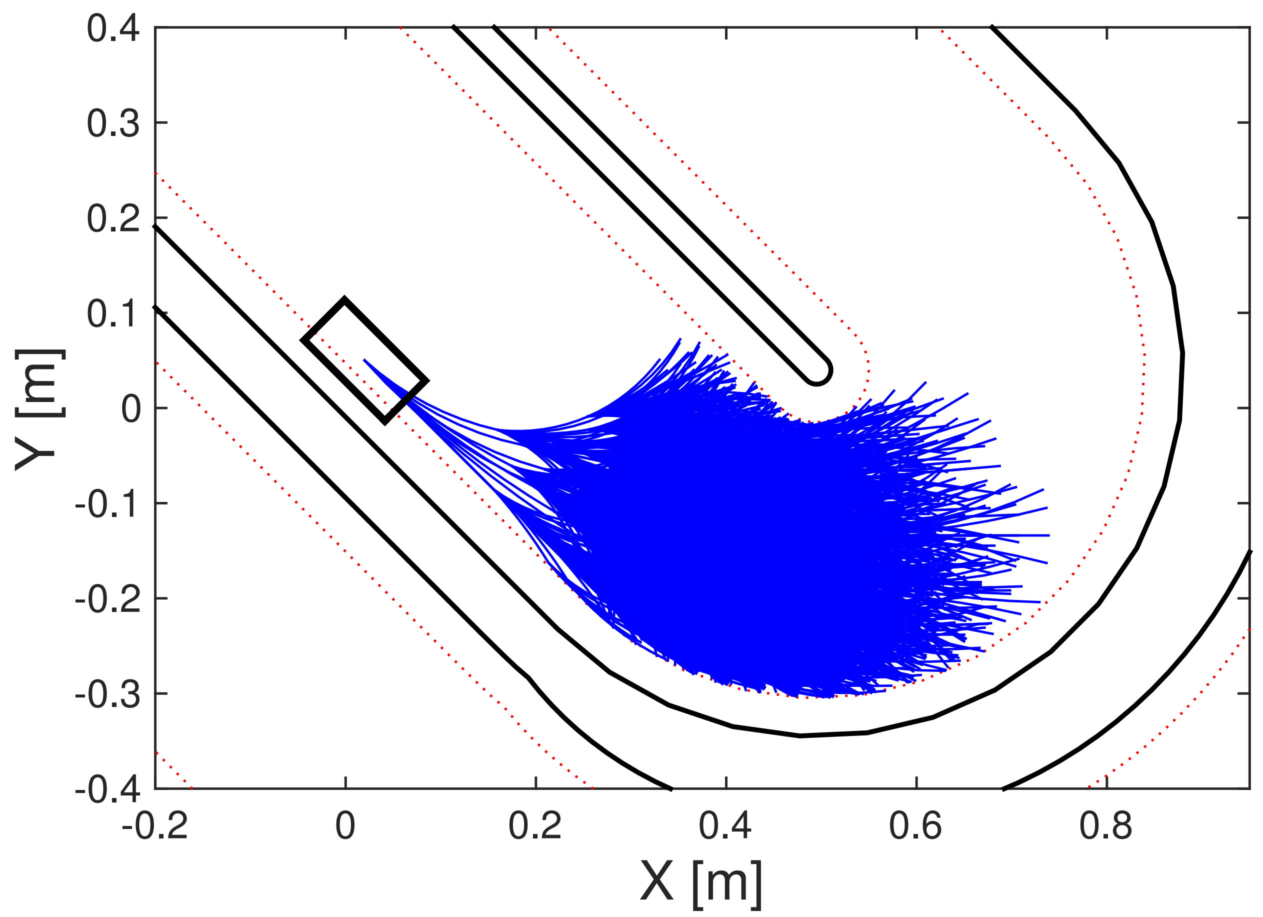
Hierarchical Control Structure

High-level motion planning based on constant velocities primitives

- Plan for slow dynamics
- Reduced dimension —> four dimensions instead of six
- Long discretization times —> 0.16s instead of 0.02s

MPC-based trajectory tracking

- Considering full dynamical bicycle model
- Linearization points given by path planner



- 129 constant velocity motion primitives
- 3 prediction steps
- Lookahead of 0.48s

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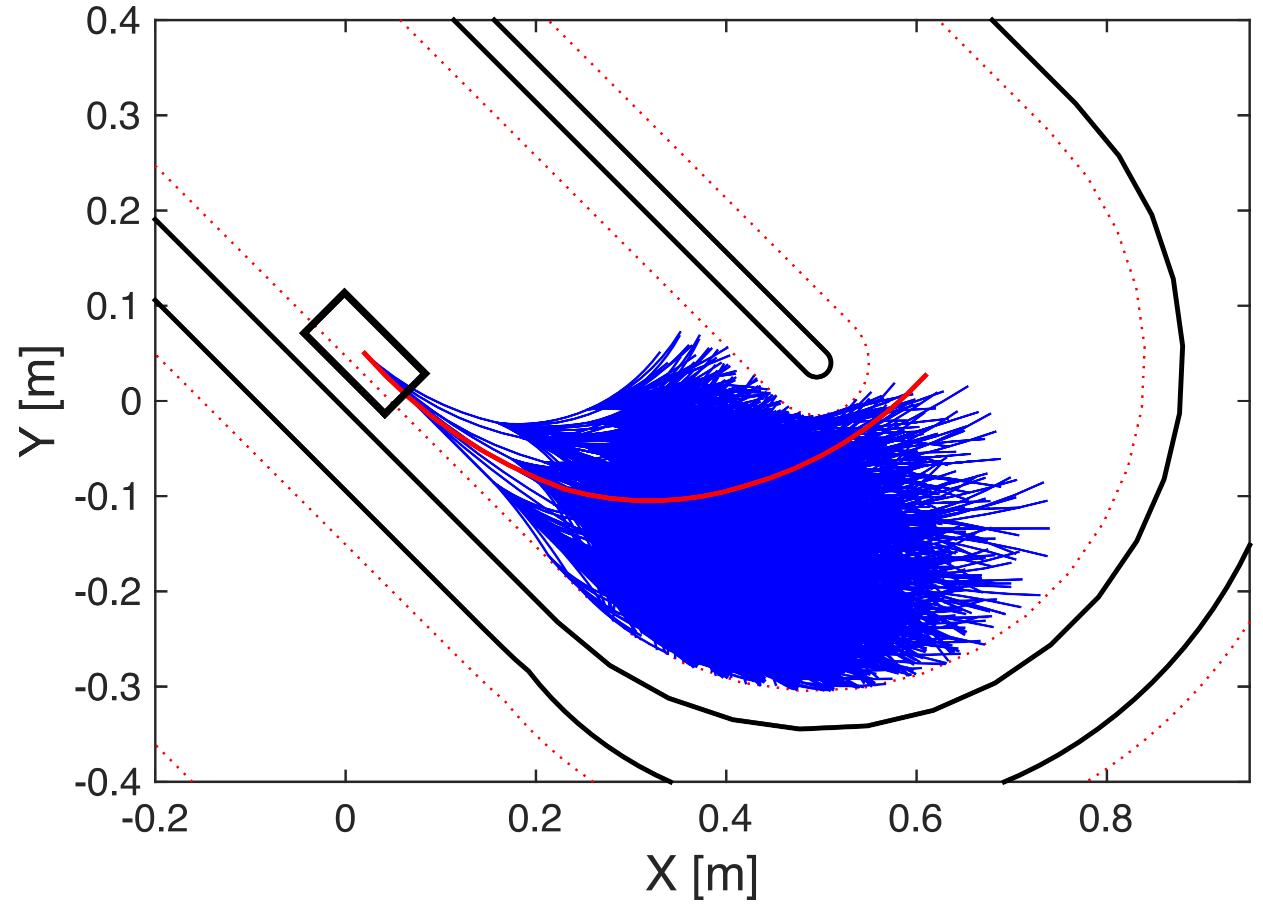

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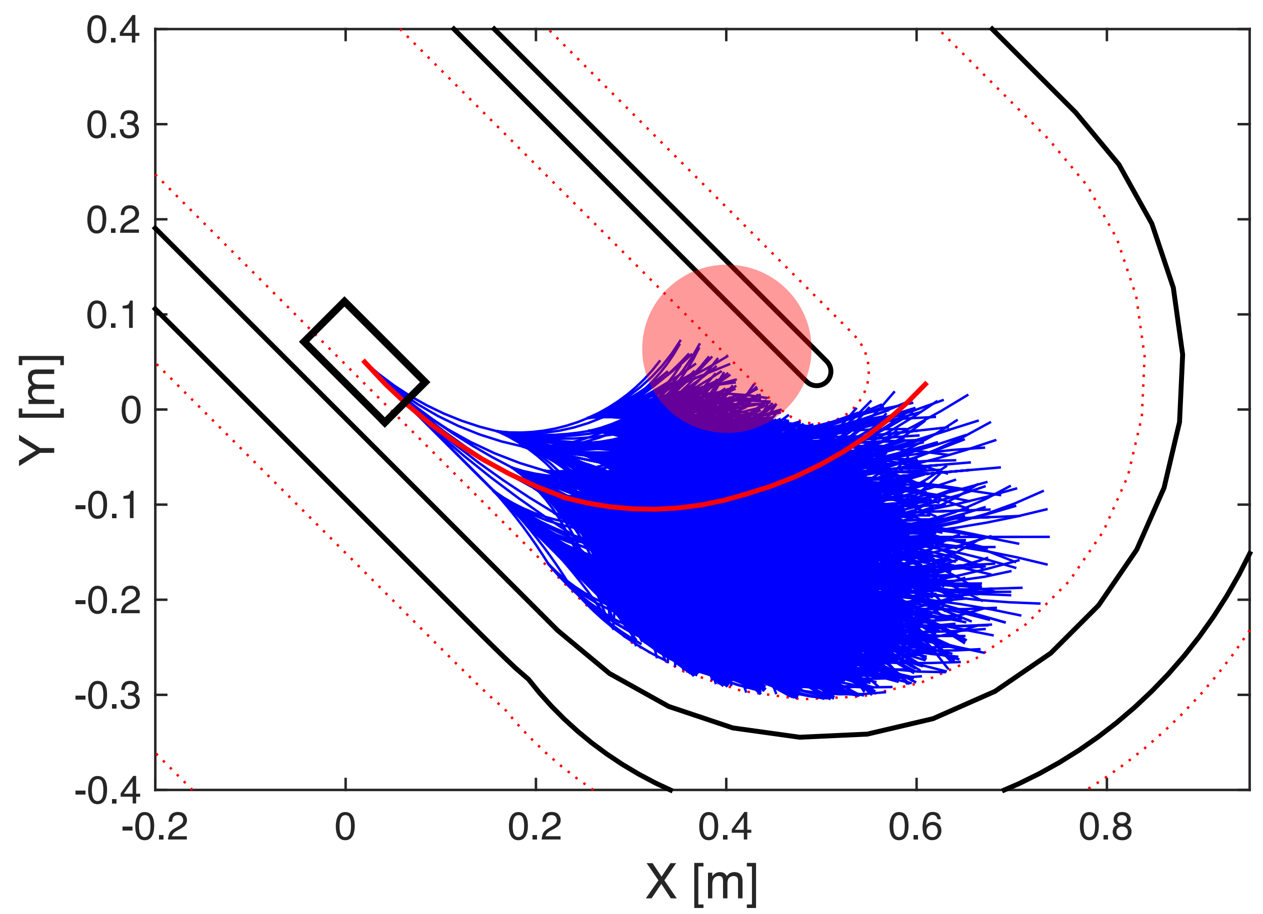

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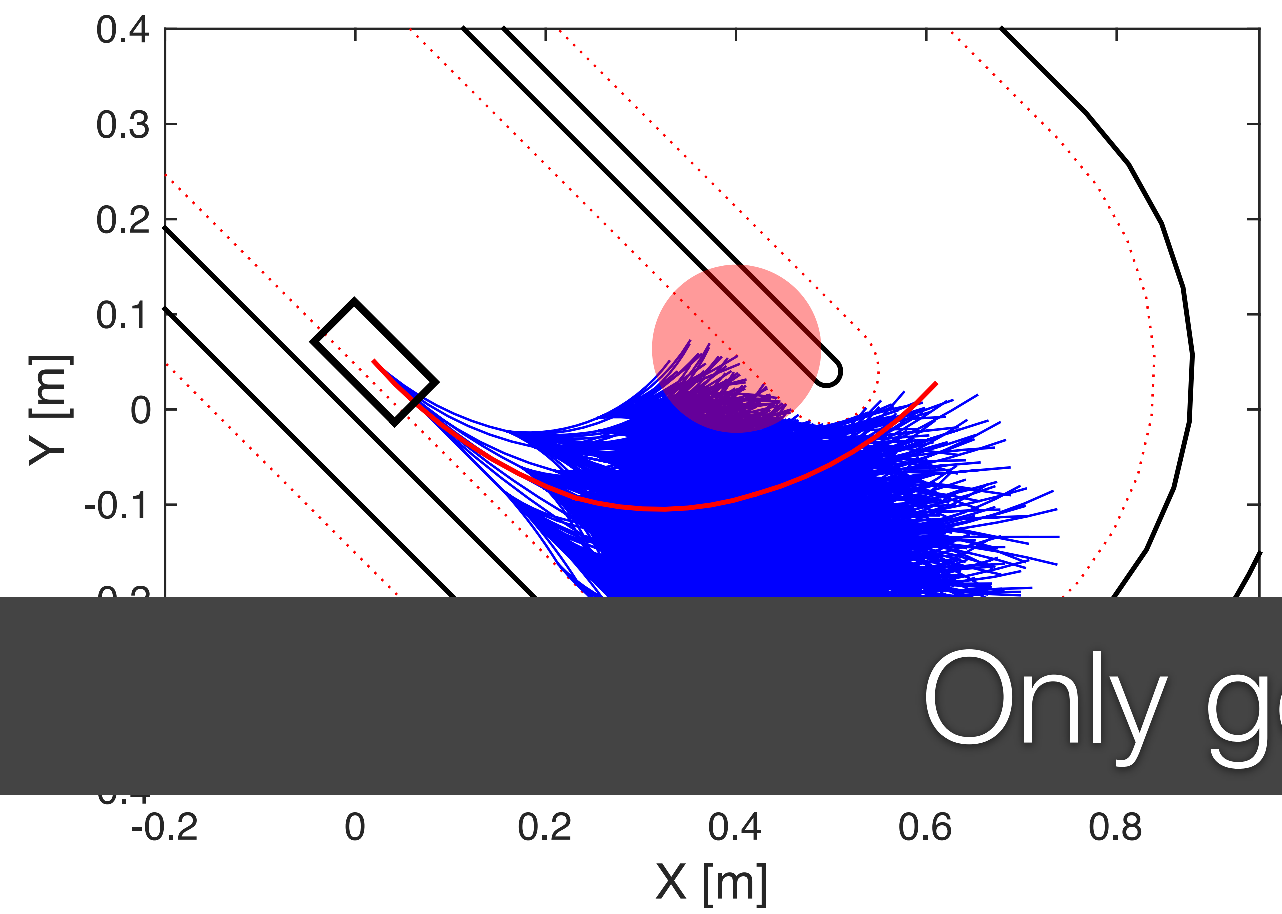

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Only generate safe trajectories

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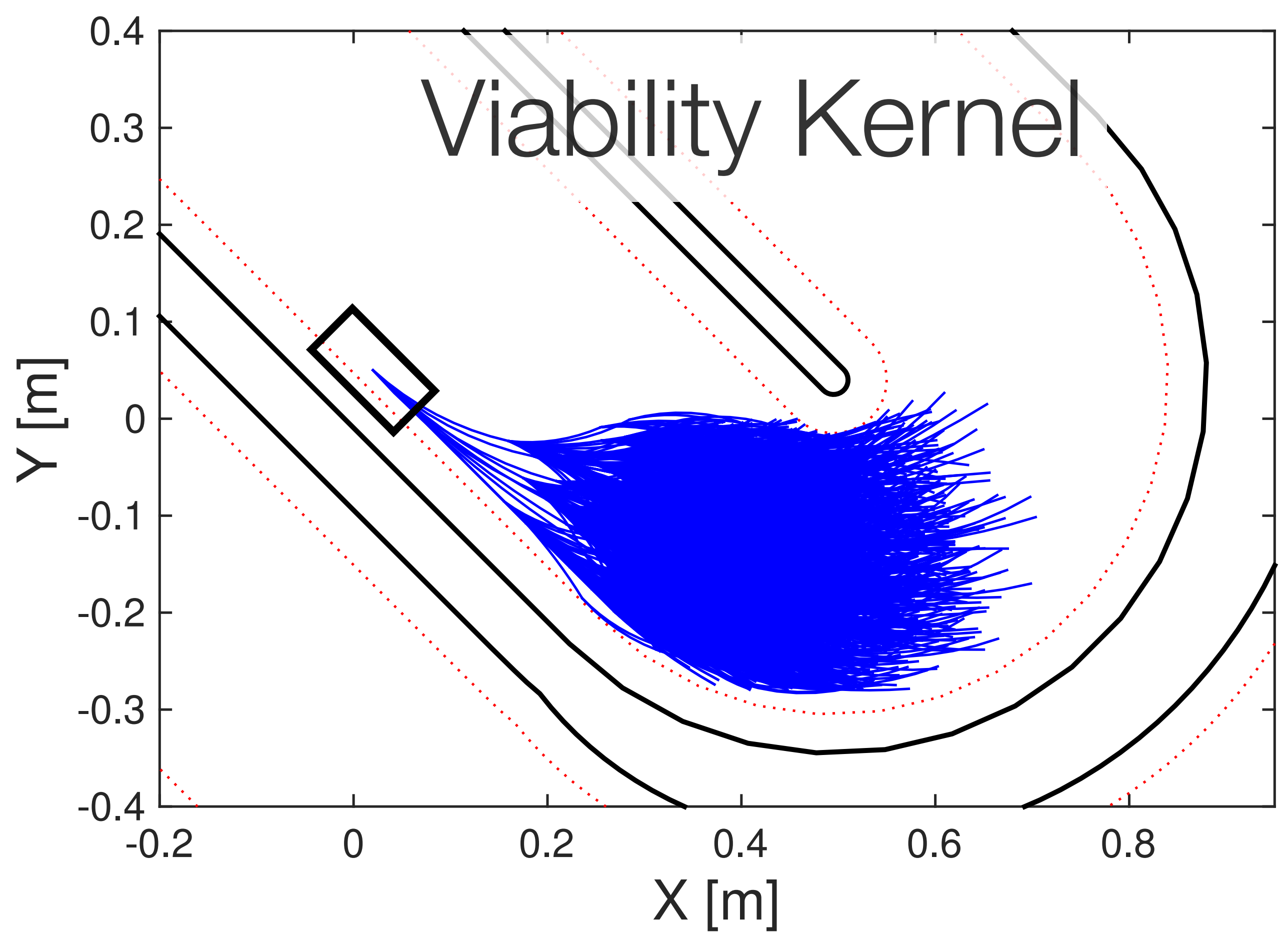
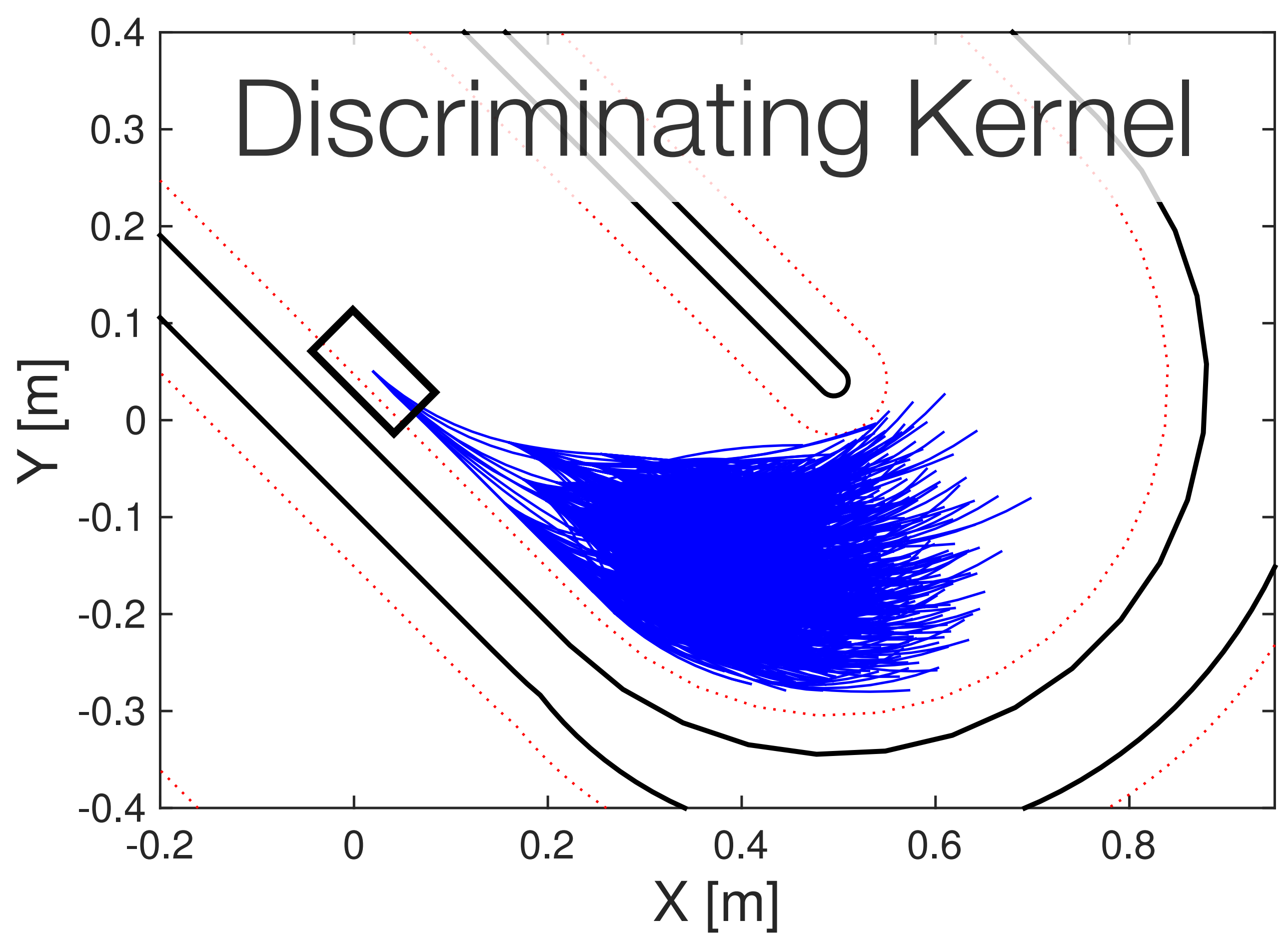
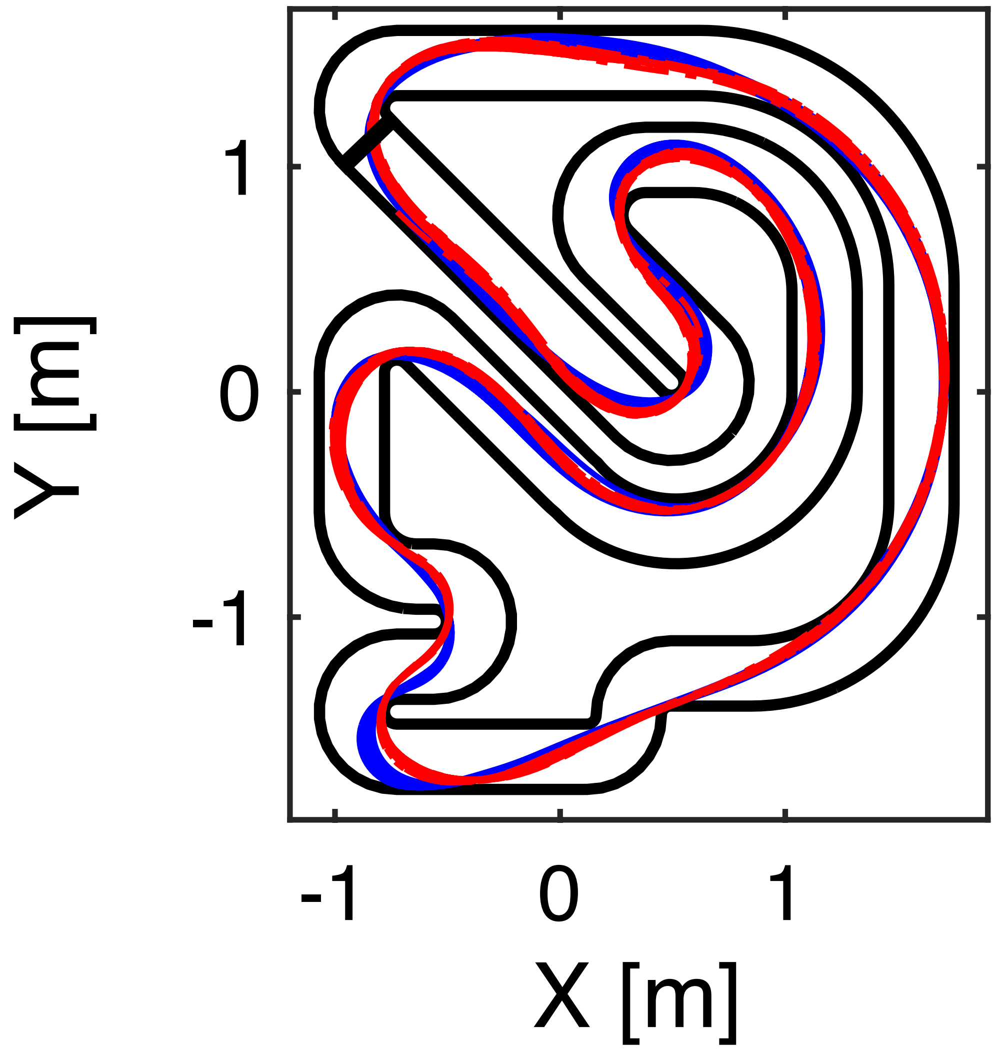
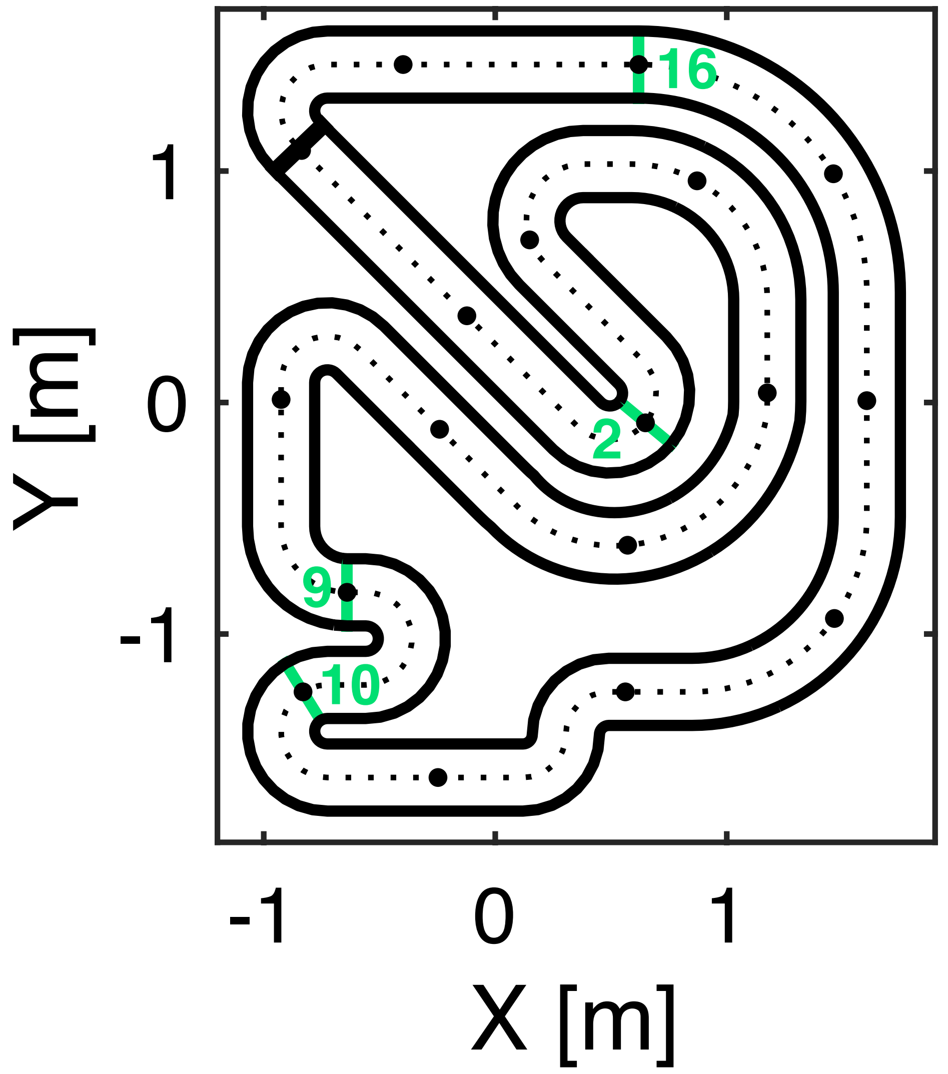


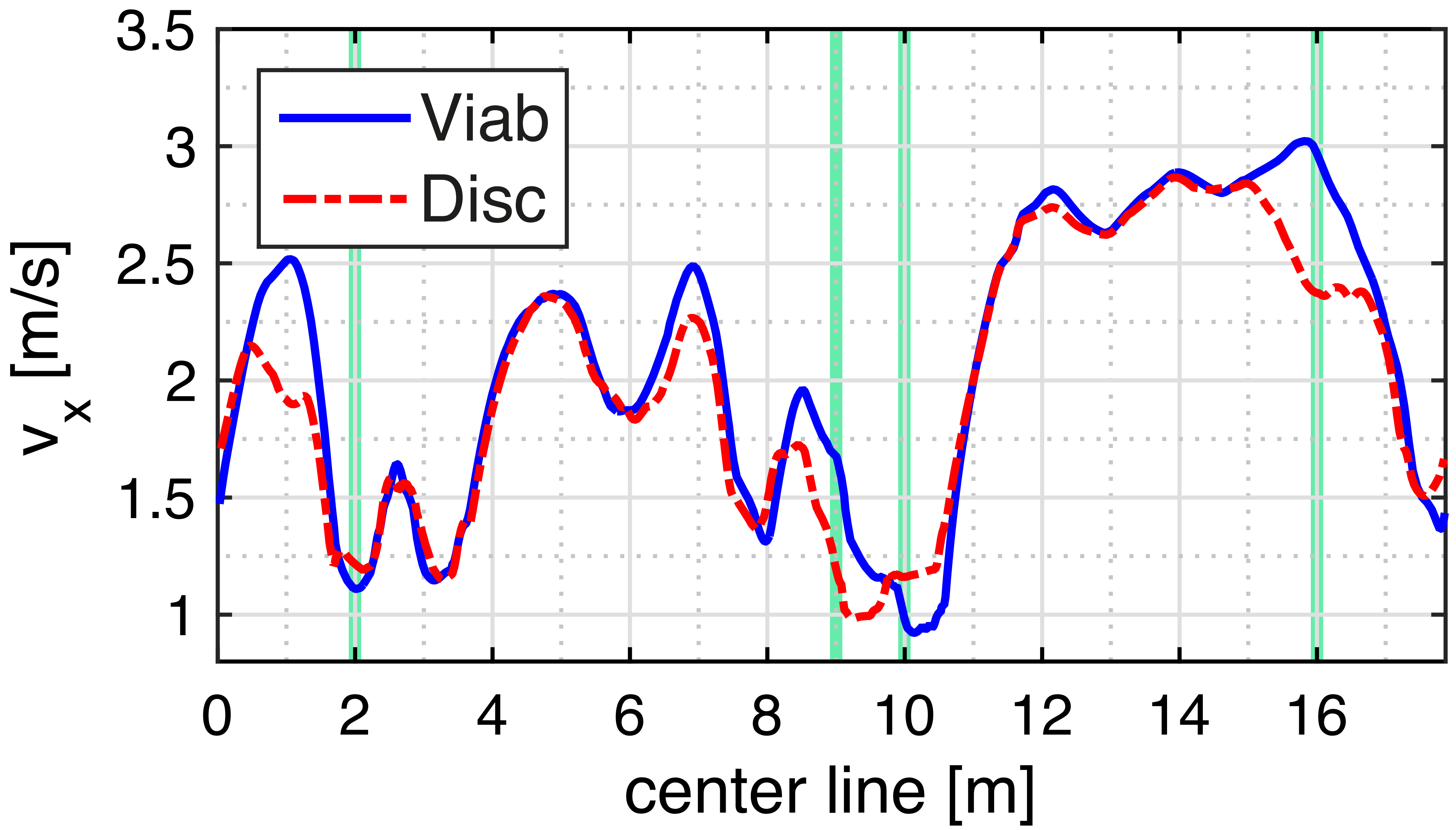


Simulation Results - Single Agent





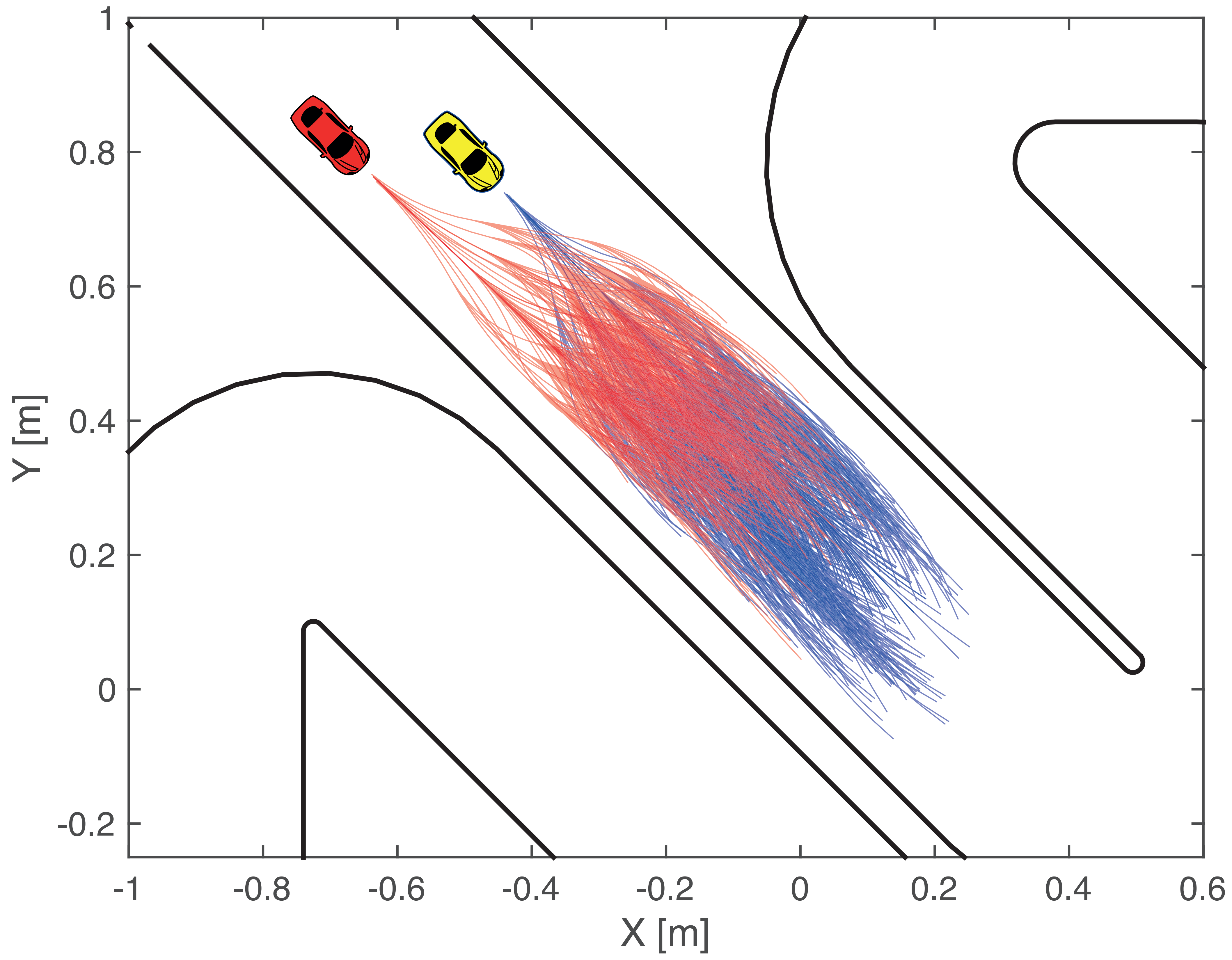
Kernel	mean lap time [s]	# constr. violations	median comp. time [ms]	max comp. time [ms]
No	8.76	4	32.26	247.7
Viab	8.57	0	0.904	7.968
Disc	8.60	1	0.870	7.533



- Both methods have the same lap time
- But use different driving styles
- Difference allows for interesting racing

Bimatrix Racing Games

- Every trajectory is an action of a car
  - Each trajectory has a payoff
  - Payoff depends on actions of both cars



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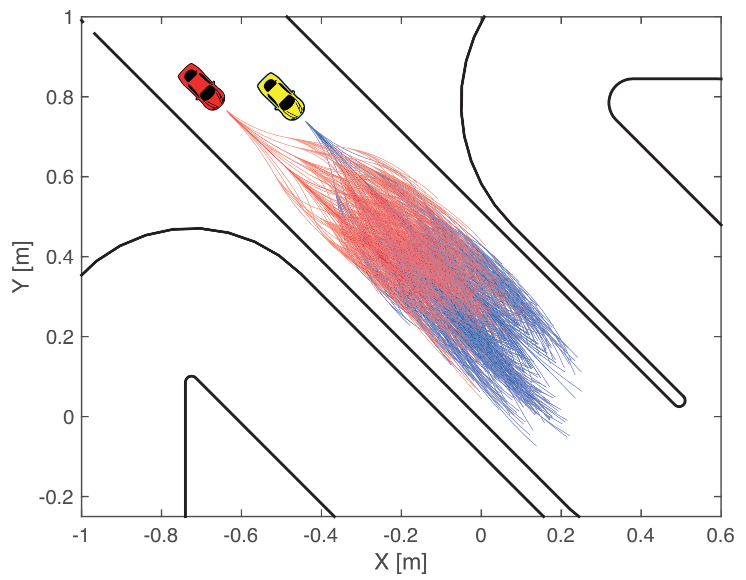

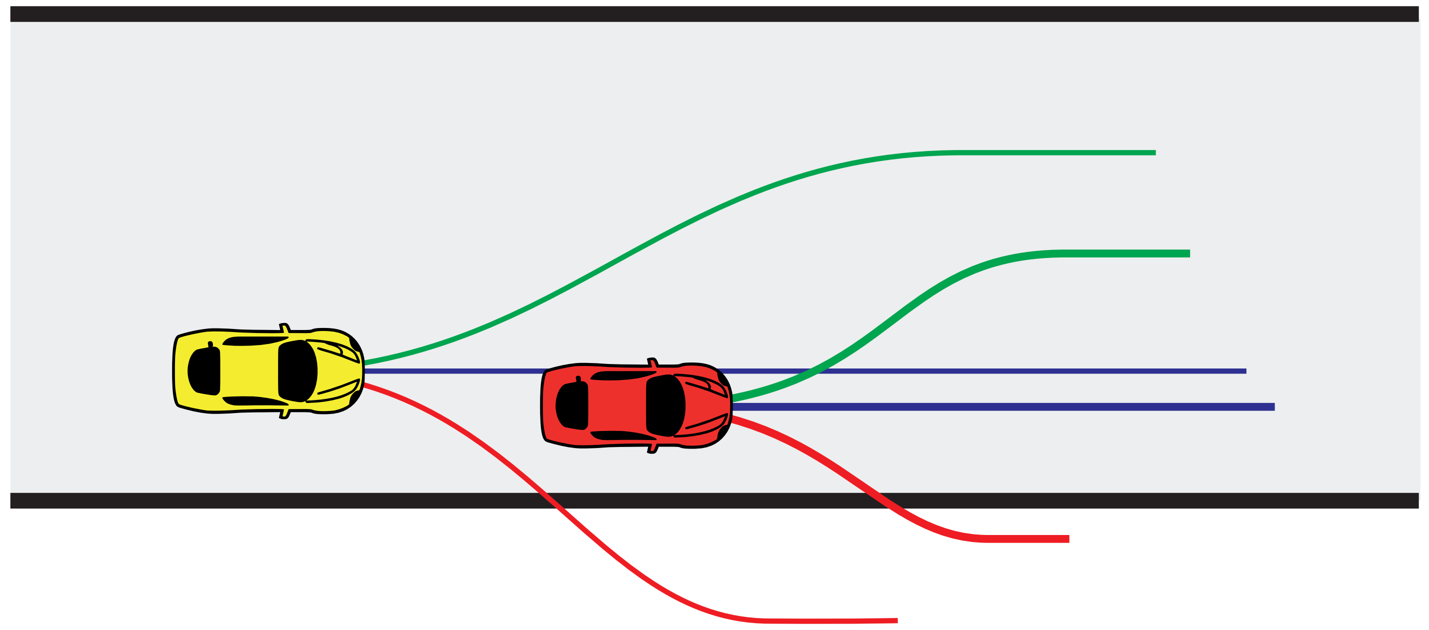

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \quad B = \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{bmatrix}$$

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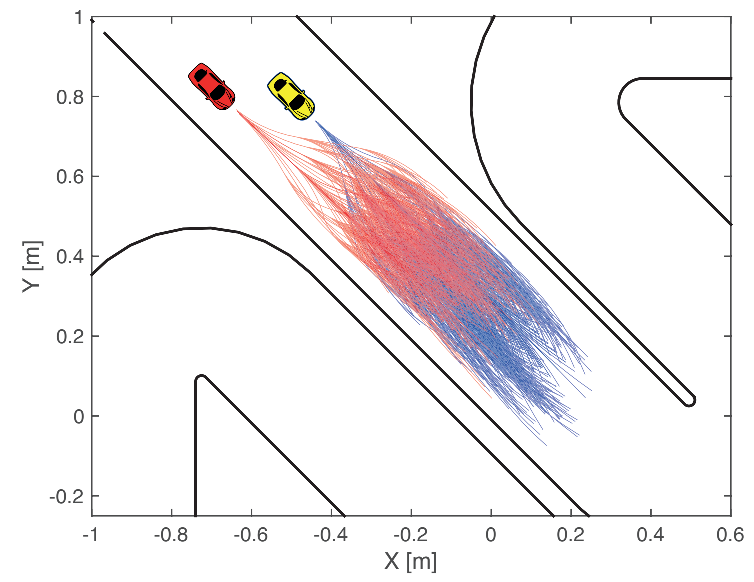

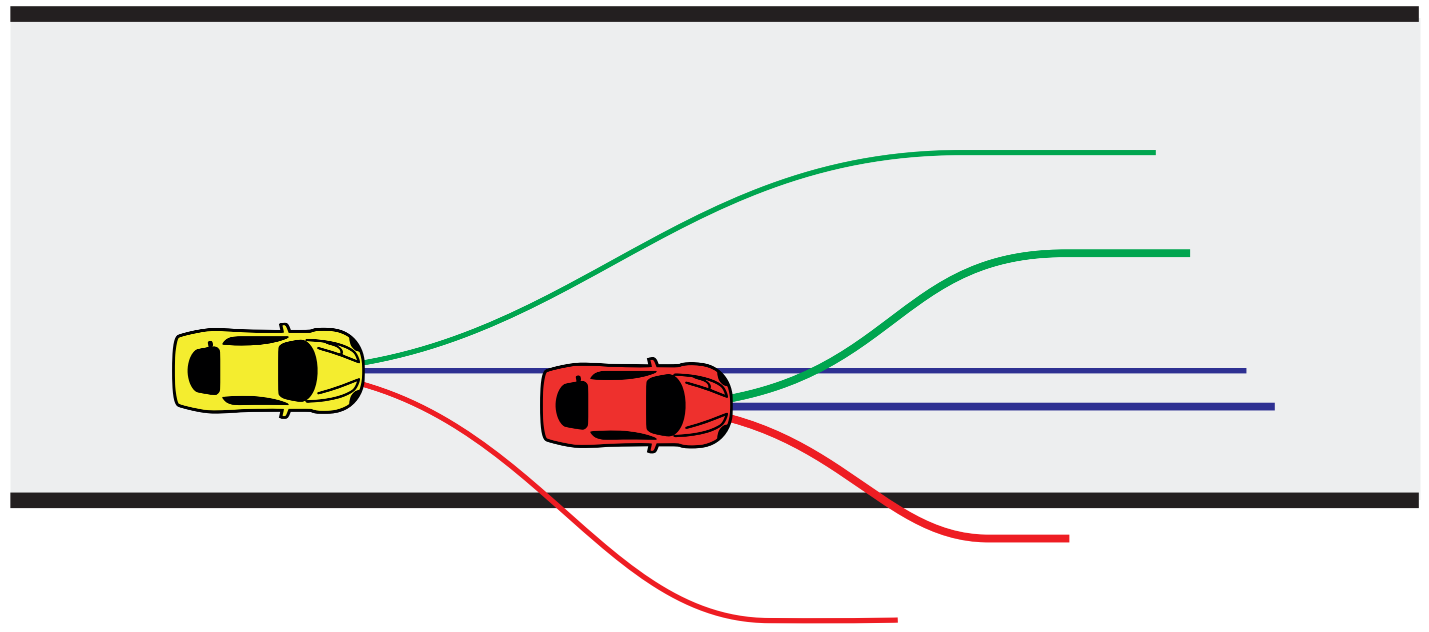

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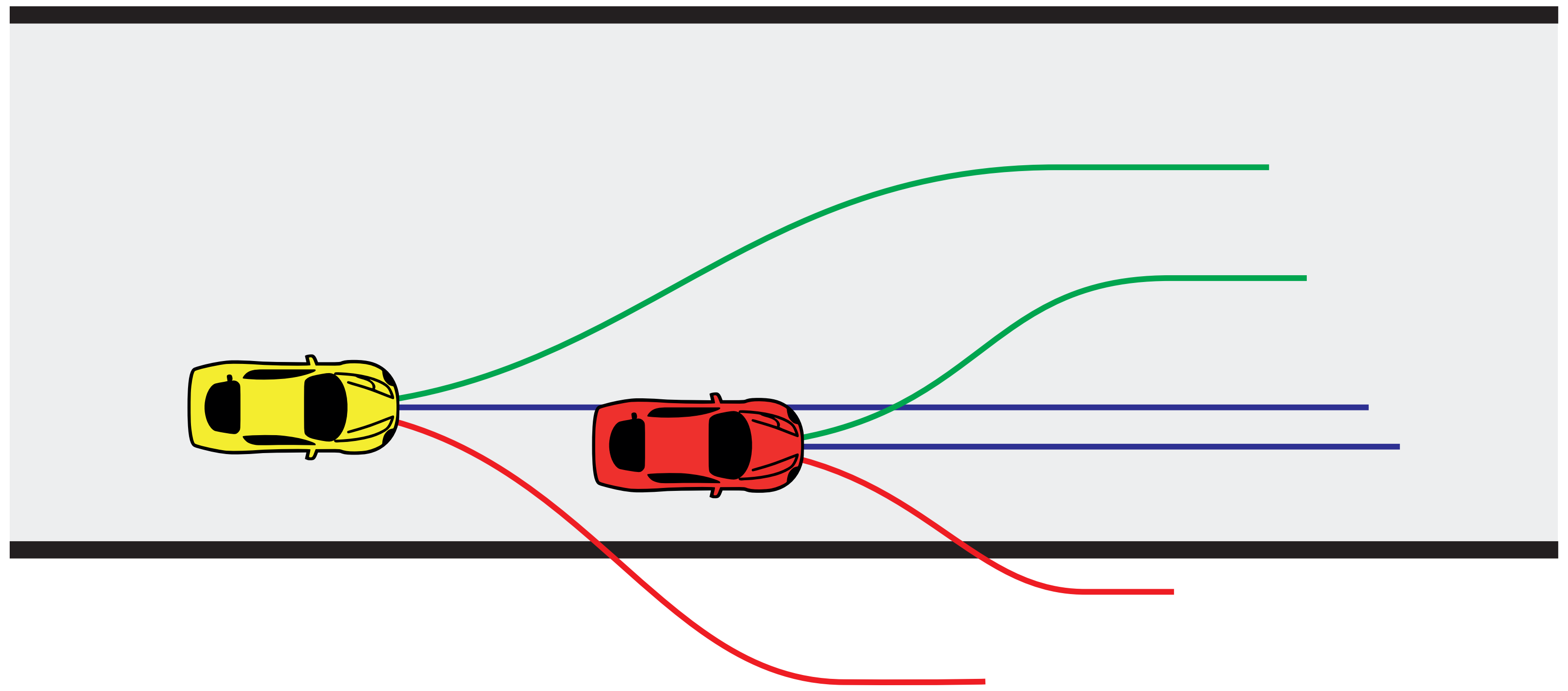
- The leader is always the car which is ahead at the beginning
- A trajectory pair is feasible if:
  - Trajectories stay inside the track and do not collide

Three Racing Games

- Sequential Game
- Cooperative Game
- Blocking Game



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- Exploiting the leader-follower structure
  - Low payoff if a trajectory leaves the track
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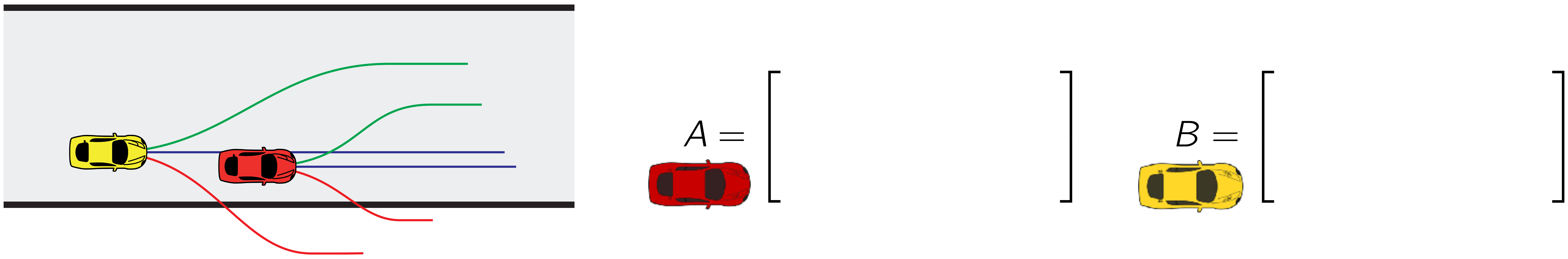
  - Low payoff for the **follower** if trajectories collide

Cooperative Game

Blocking Game



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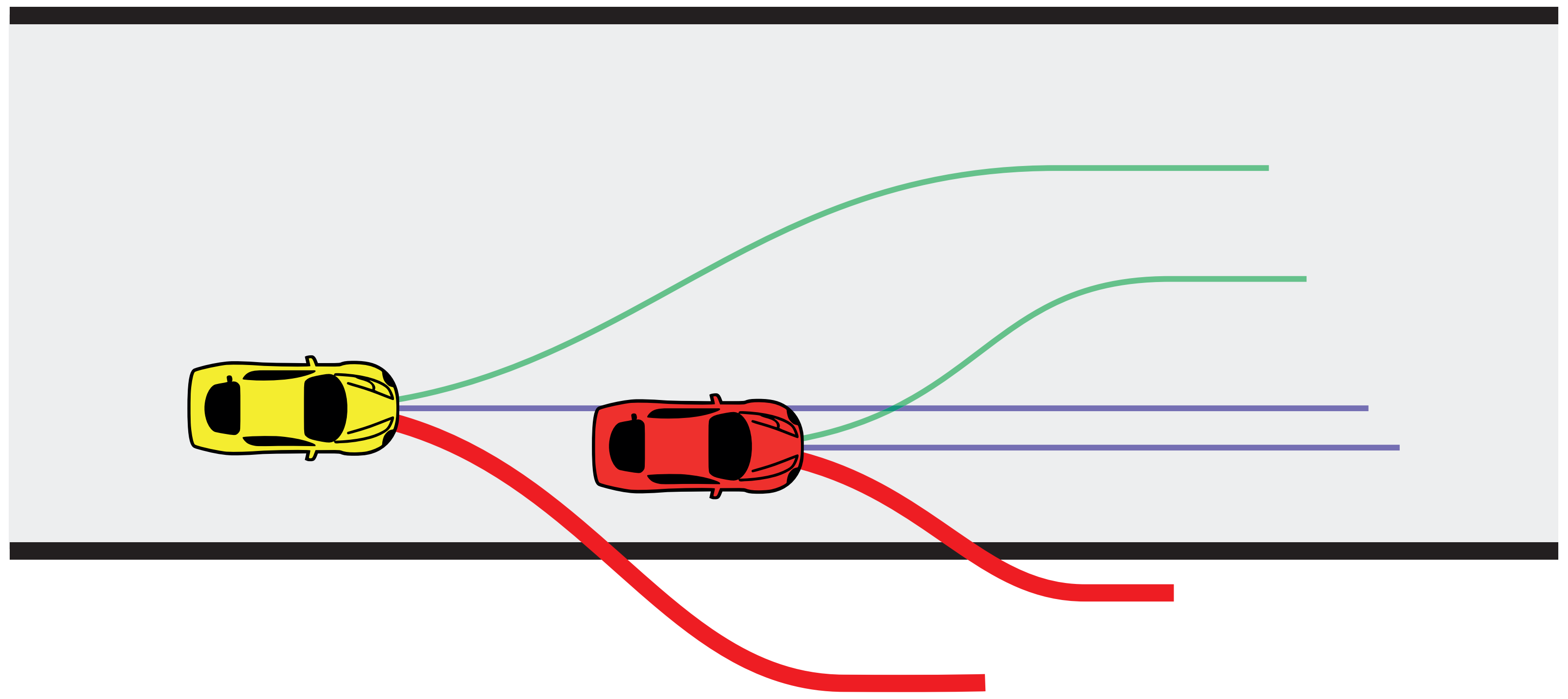

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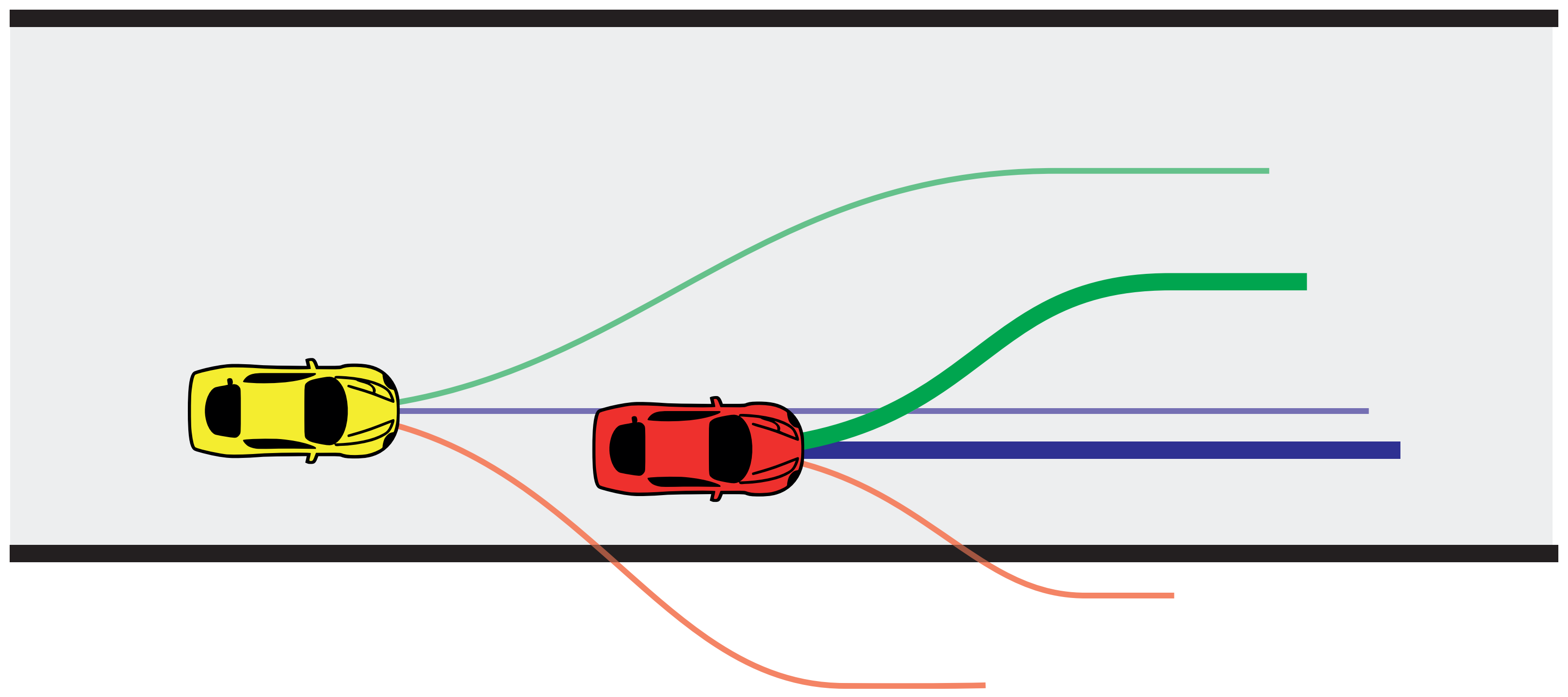

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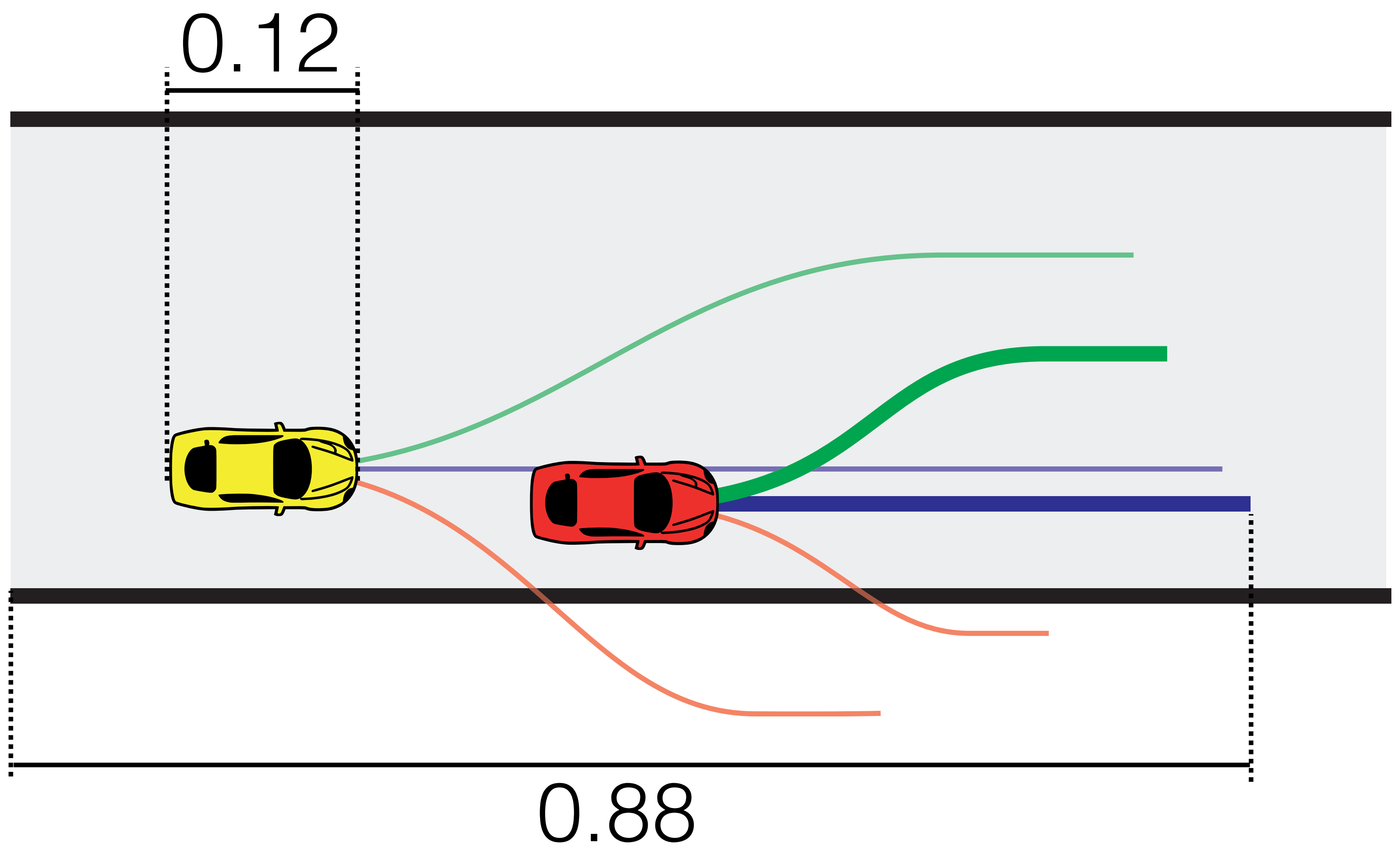

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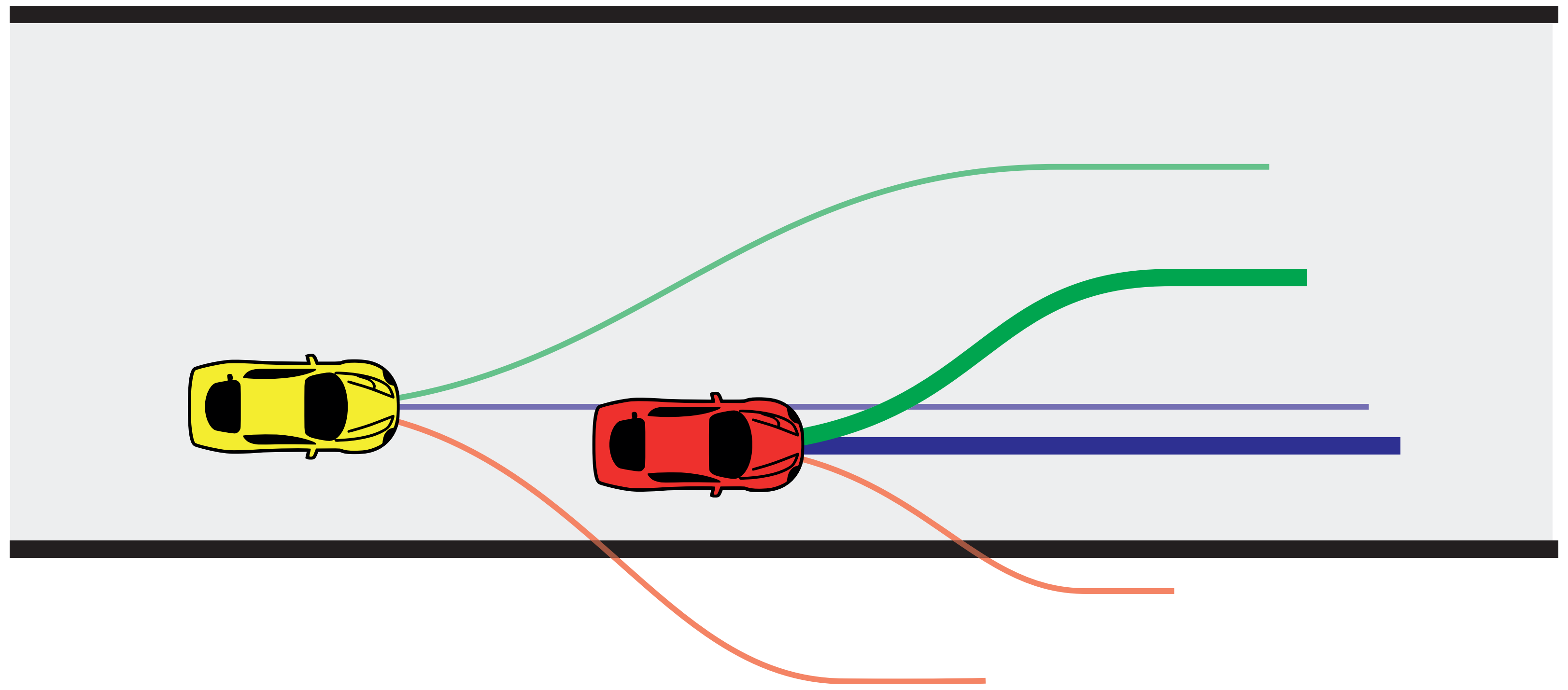

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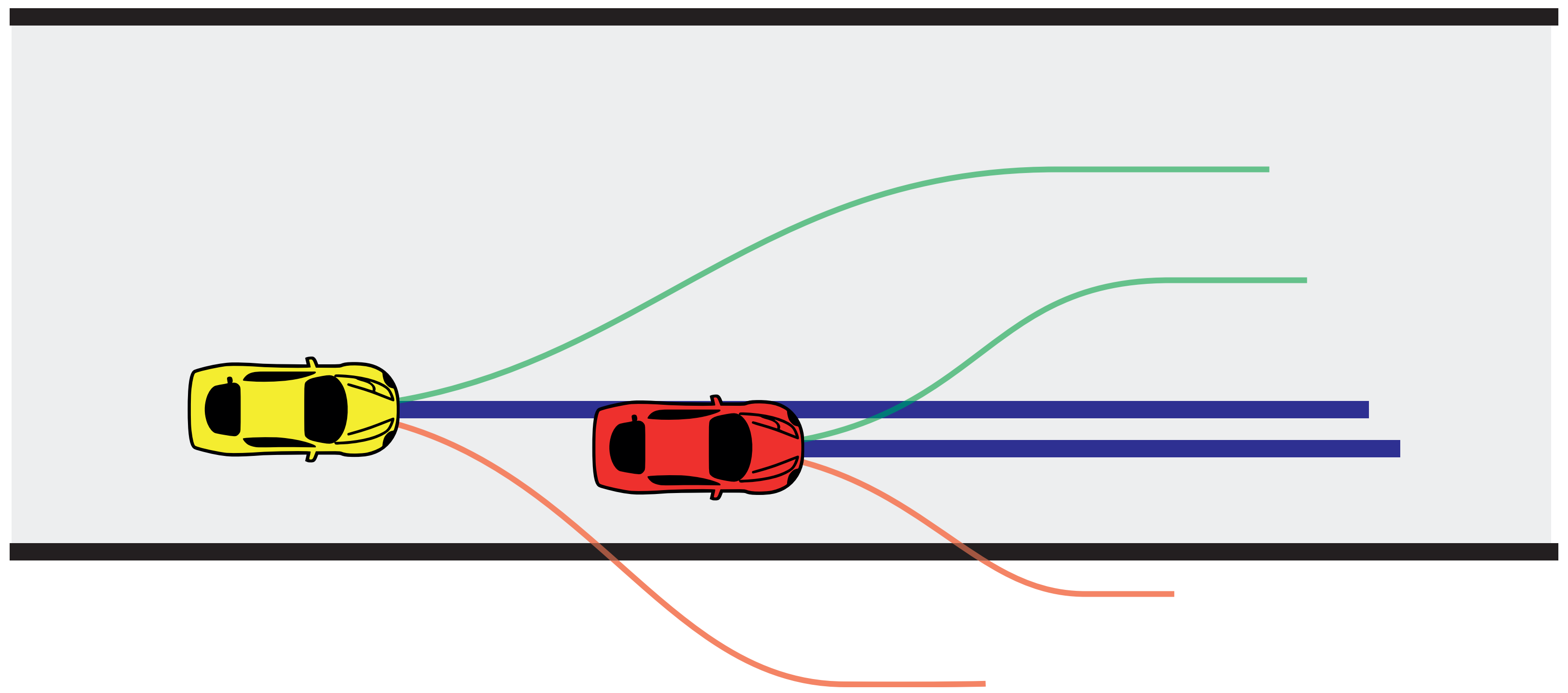

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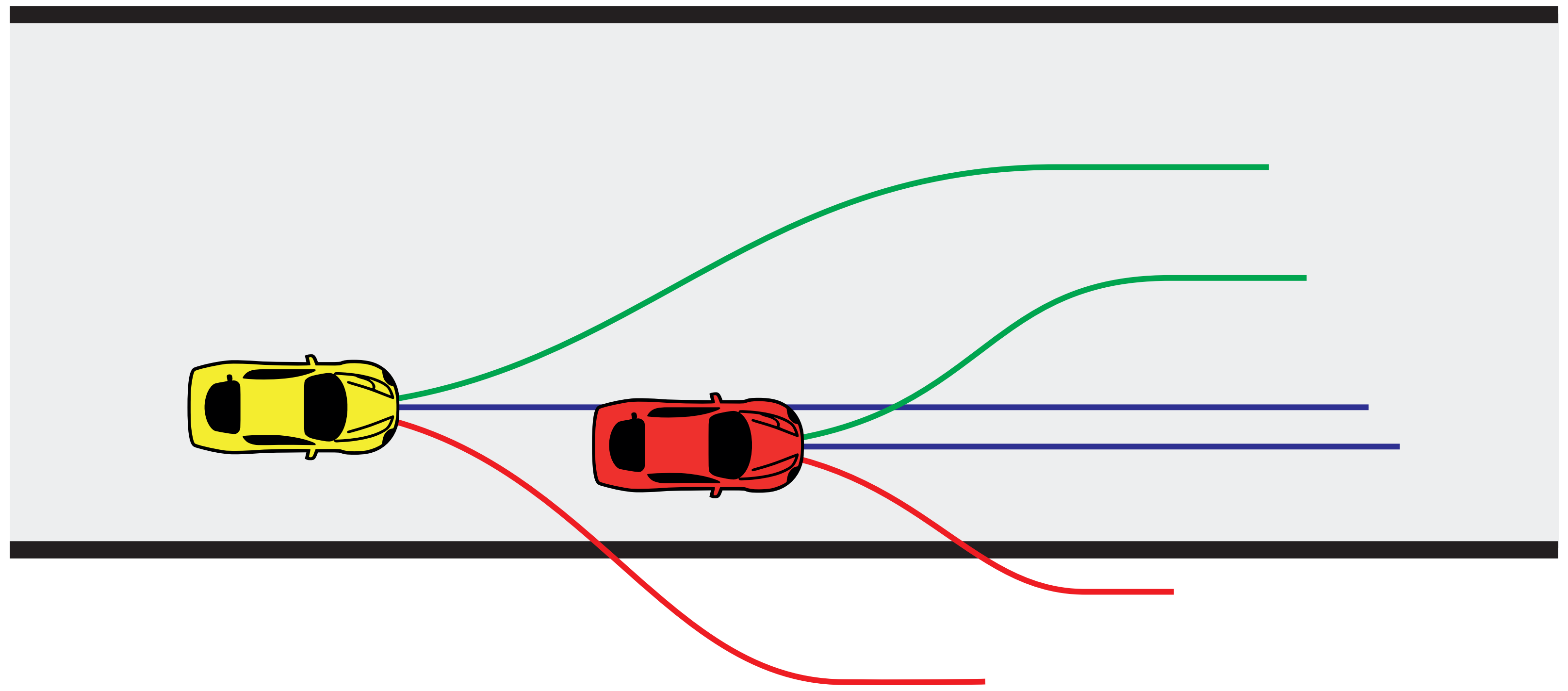
Three Racing Games

Sequential Game

Cooperative Game

- Both cars consider collisions
  - Low payoff if a trajectory leaves the track
  - Low payoff if the trajectories collide
  - Progress payoff if a trajectory is feasible

Blocking Game



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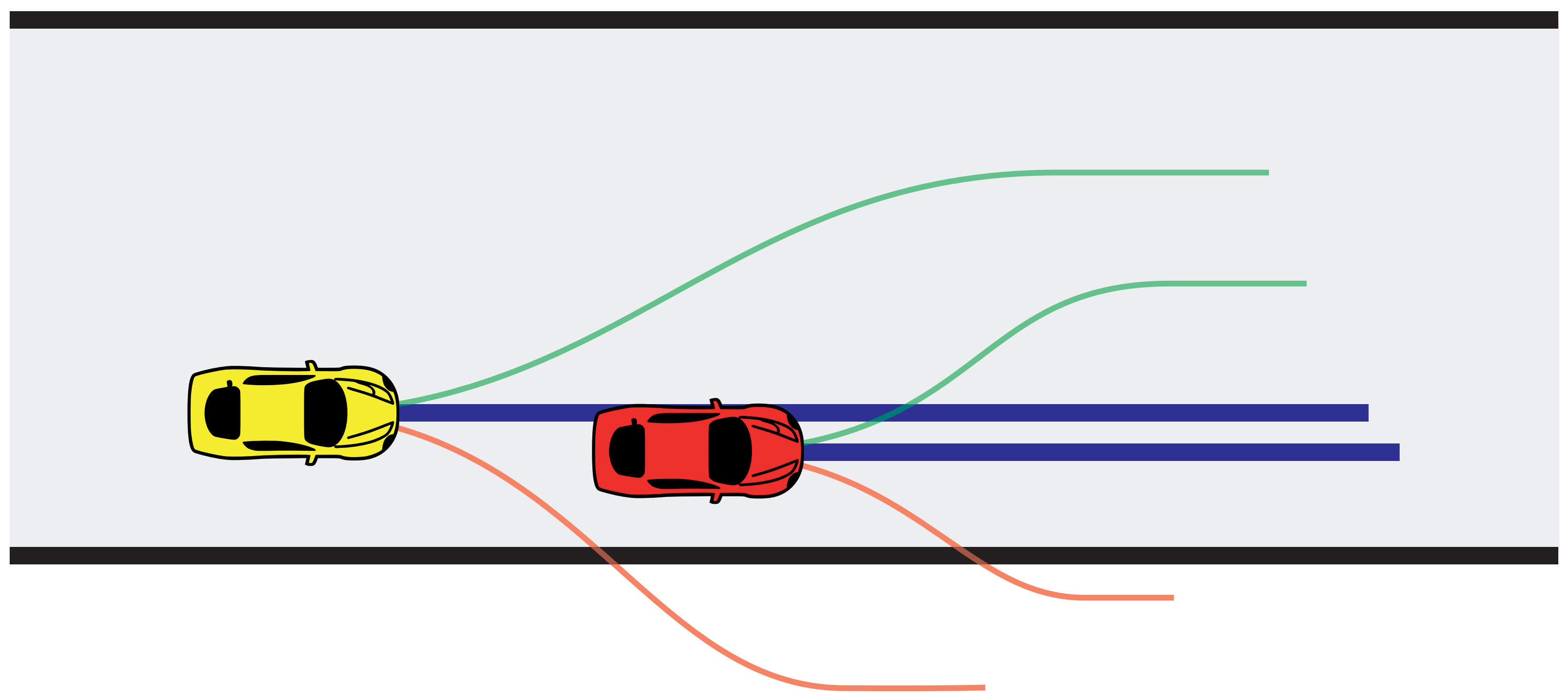

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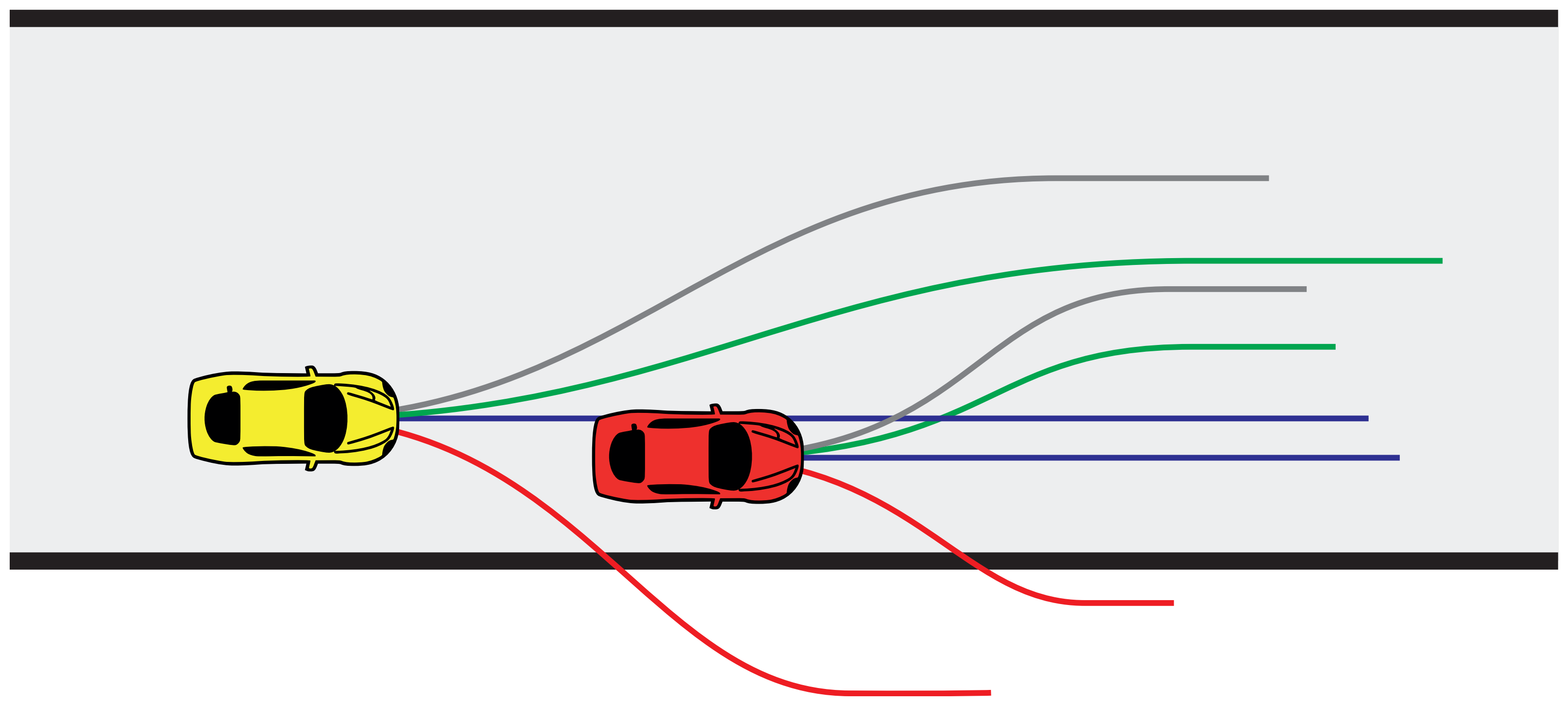
- Additional reward for staying **in front** at the end of the horizon



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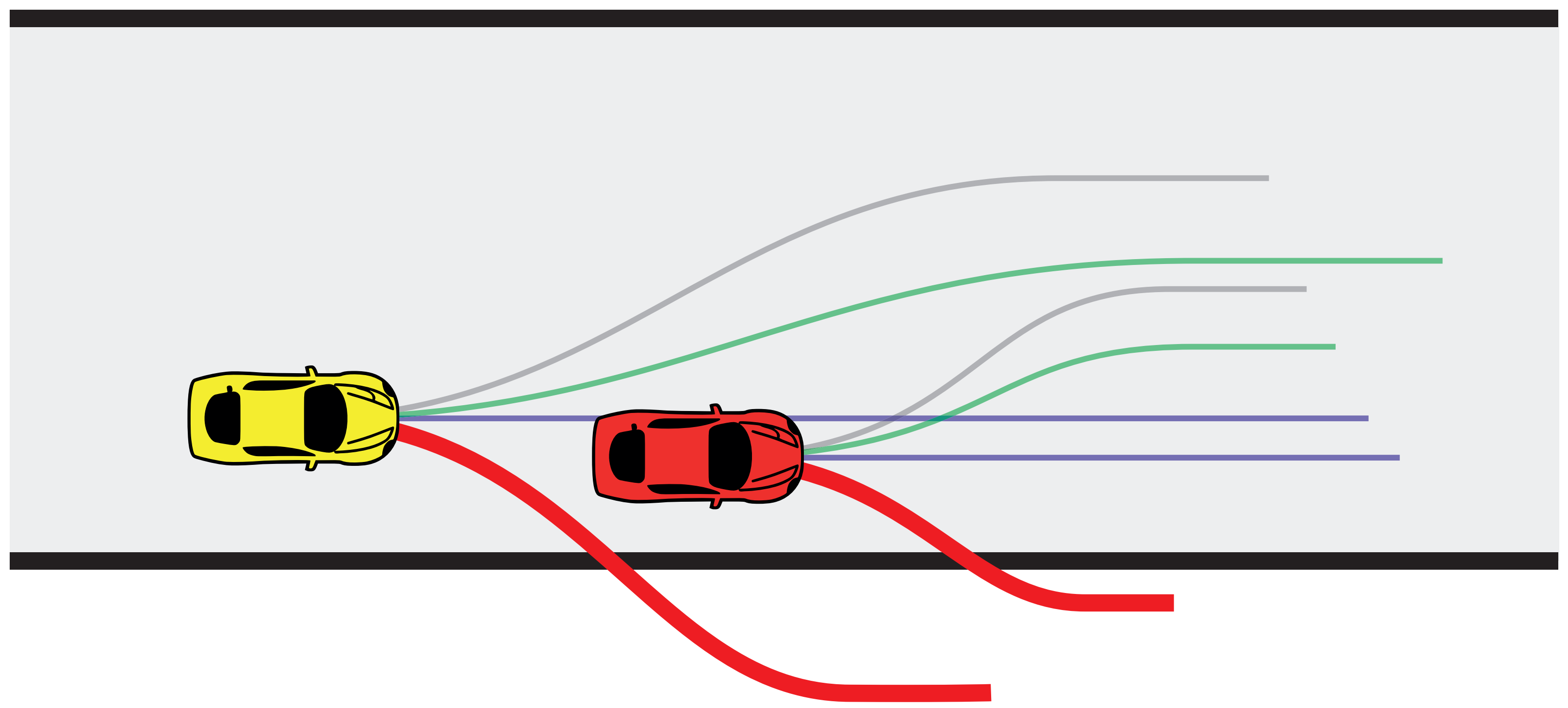

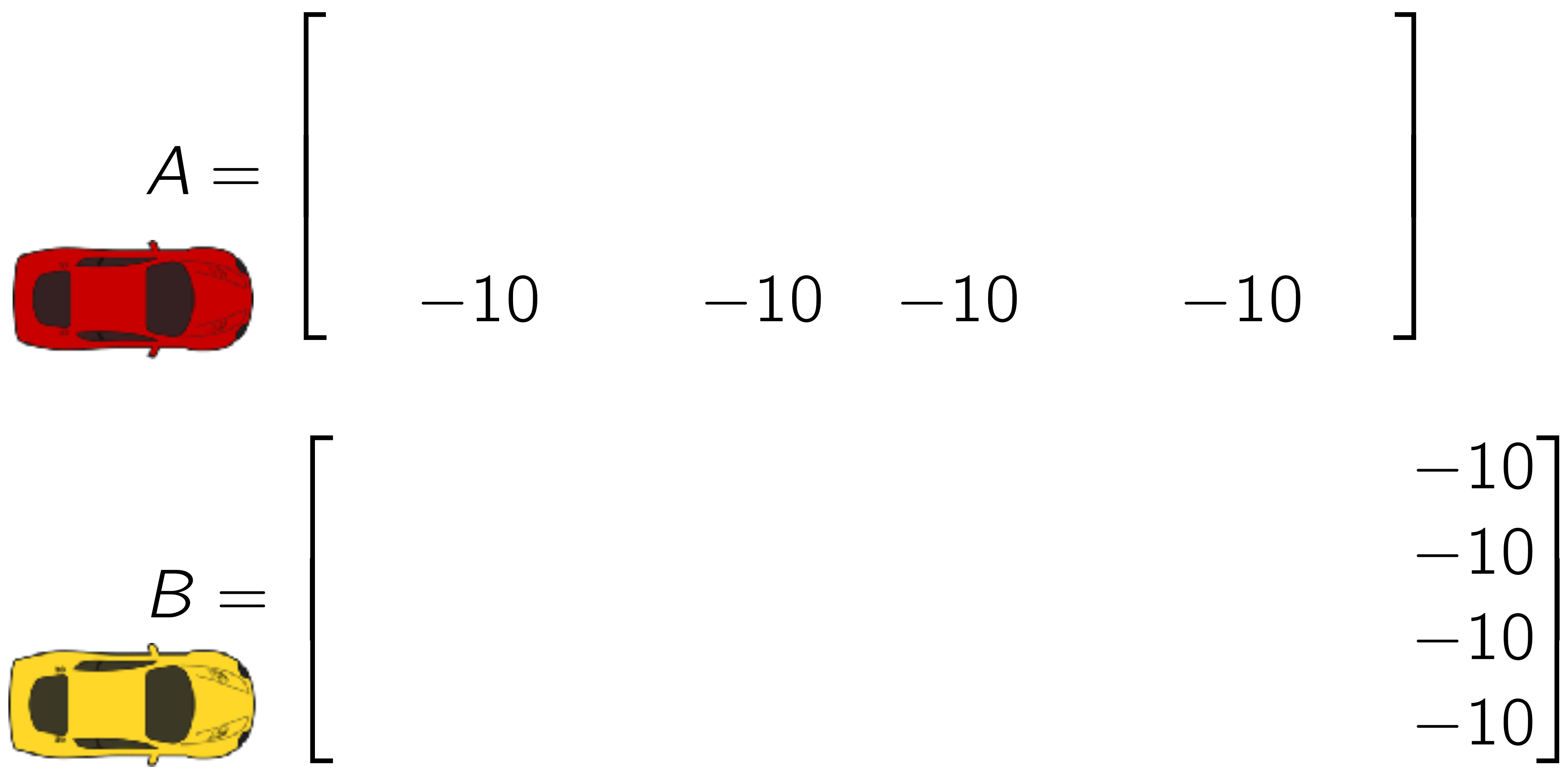

Three Racing Games

Sequential Game

Cooperative Game

Blocking Game

- Same collision structure as the cooperative game, **but:**
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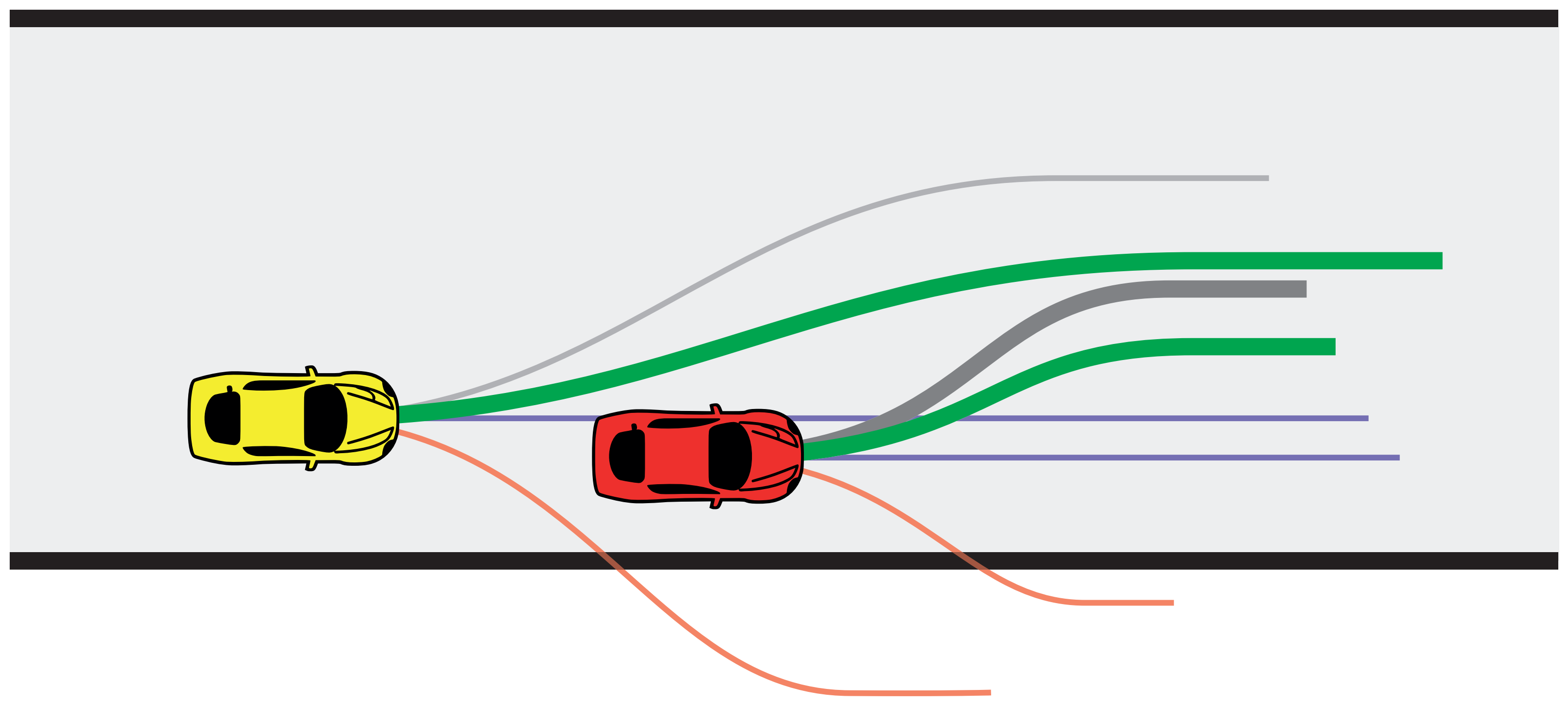

$$A = \begin{bmatrix} & -1 & & \\ & -1 & & \\ & & & \\ -10 & -10 & -10 & -10 \end{bmatrix}$$

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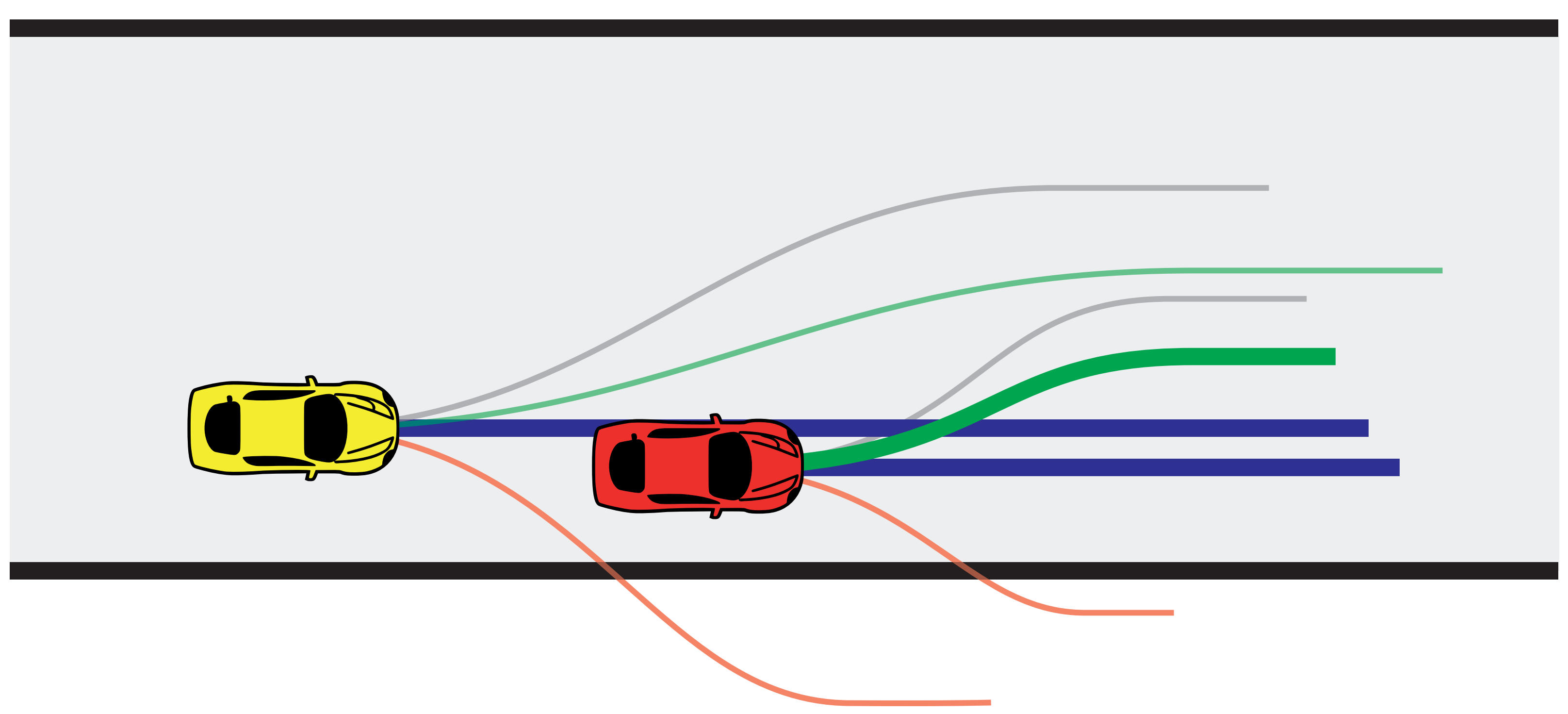

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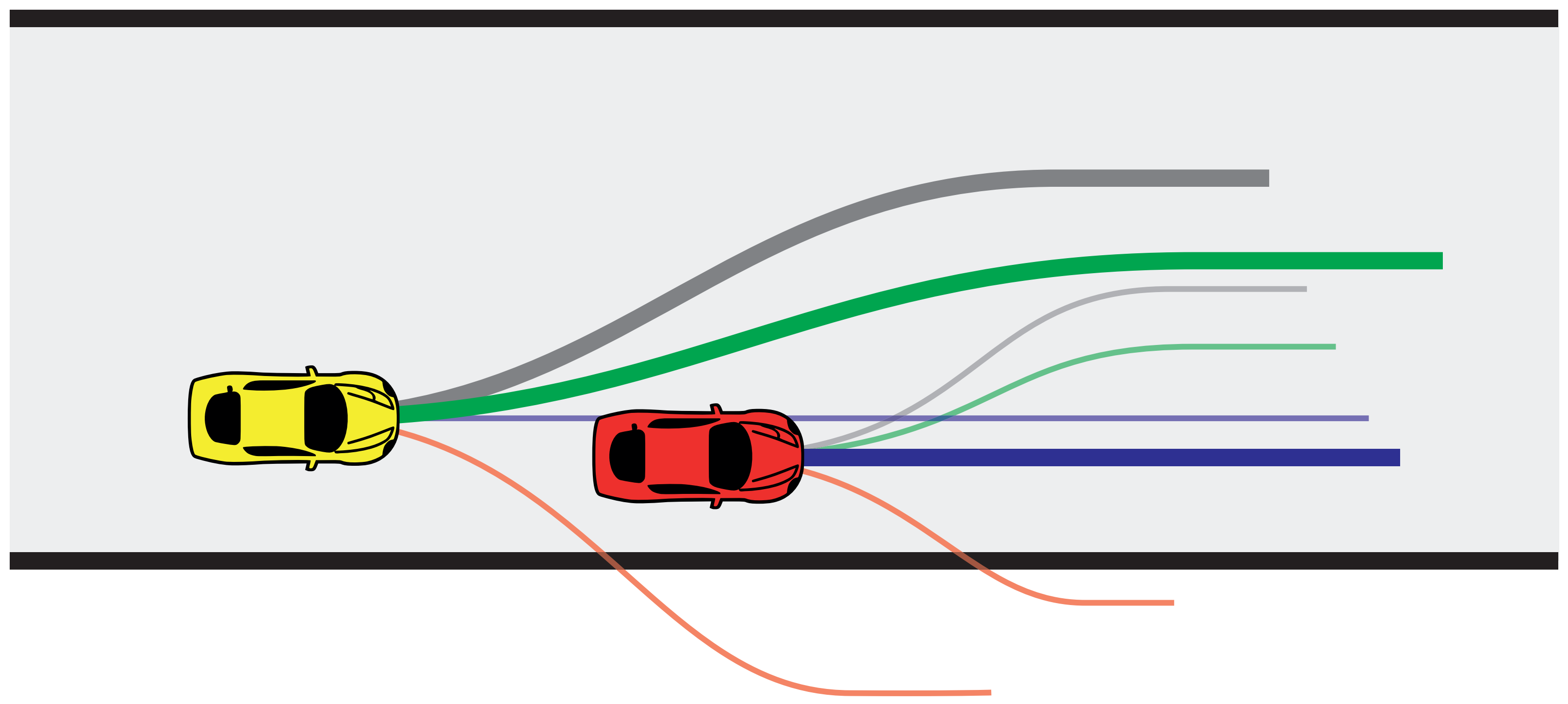

$$A = \begin{bmatrix} & & -1 & & & \\ & & -1 & -1 & & \\ 0.88 & & 0.88 & -1 & 0.88 & \\ & -10 & -10 & -10 & & -10 \end{bmatrix}$$

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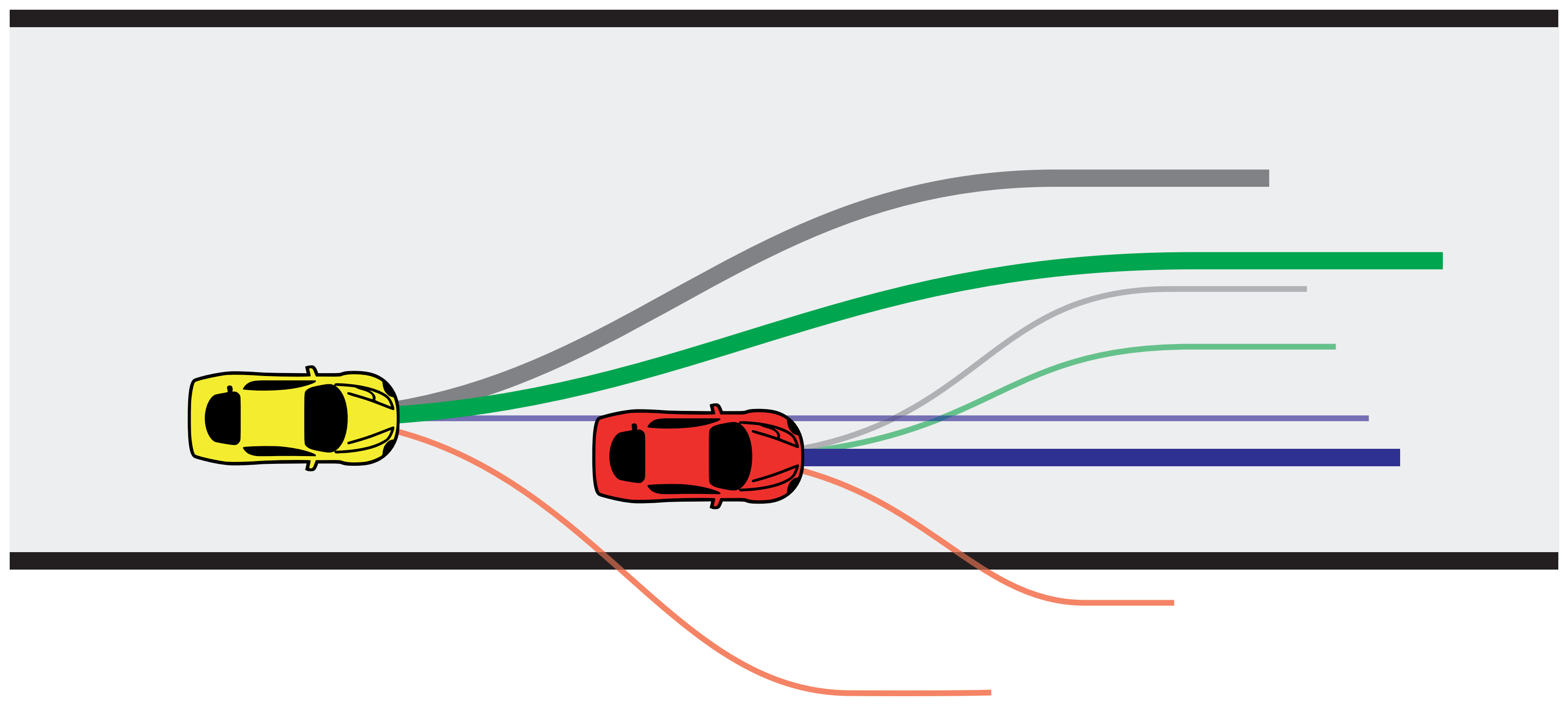

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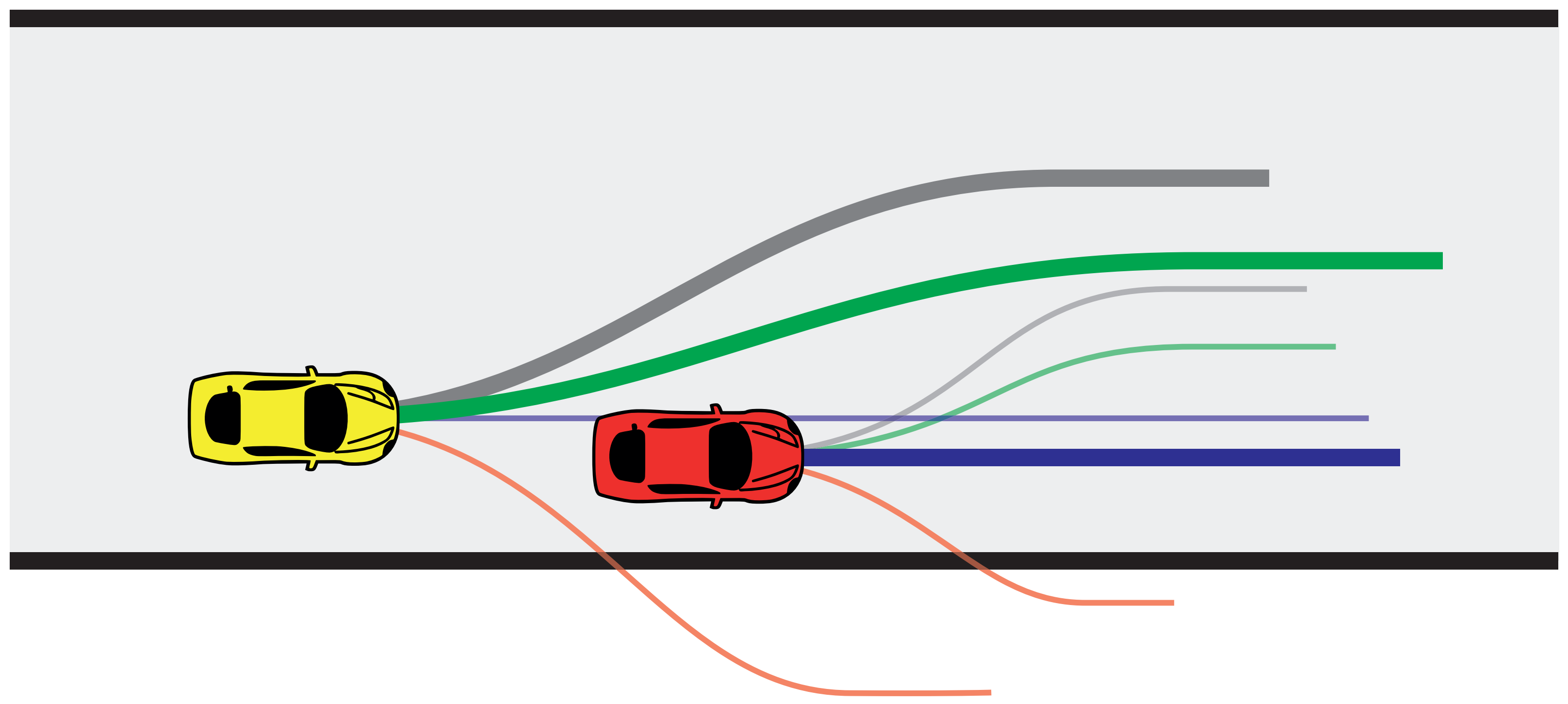

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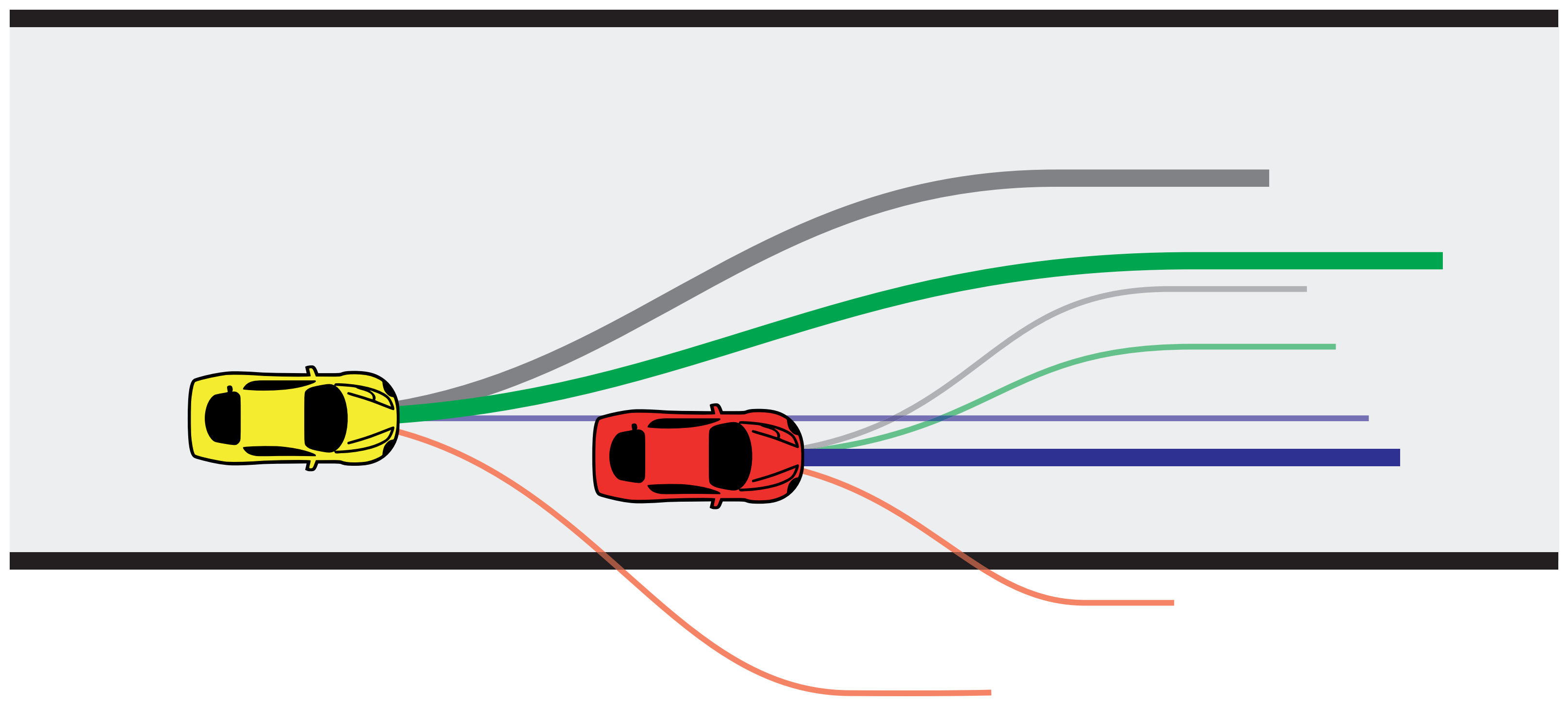

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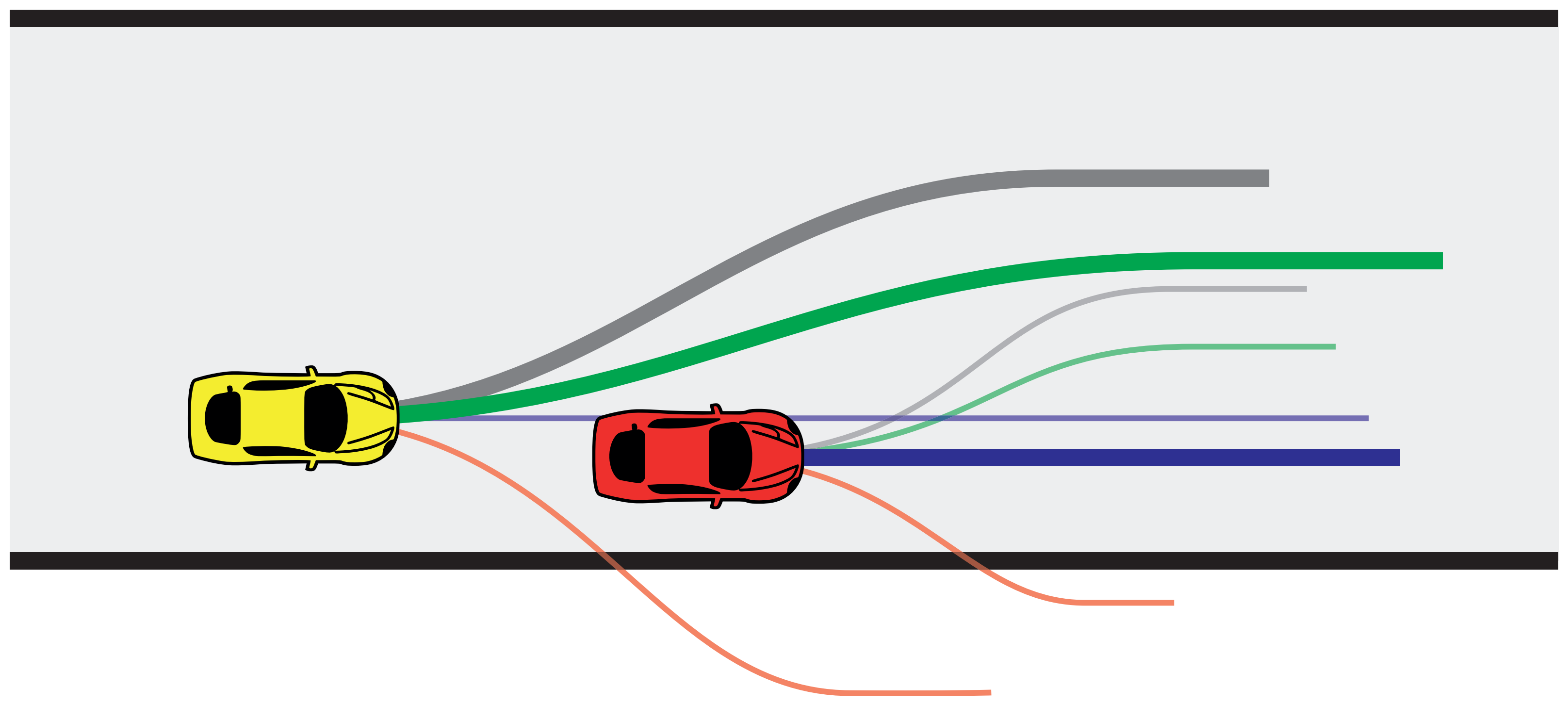
How should a car choose a trajectory?



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Equilibria concepts

- Find an equilibrium trajectory pair of the bimatrix game
  - Pure strategies (no mixed strategies)
  - $(i^*, j^*) \in \Gamma^1 \times \Gamma^2$ is an equilibrium trajectory pair

Stackelberg Equilibria

- Game with leader-follower structure
  - Leader can enforce his trajectory on the follower
  - Follower plays the **best response**: $R(i) = \arg\max_{j \in \Gamma^2} b_{i,j}$

$$i^* = \arg\max_{i \in \Gamma^1} \min_{j \in R(i)} a_{i,j}$$

$$j^* = R(i^*)$$

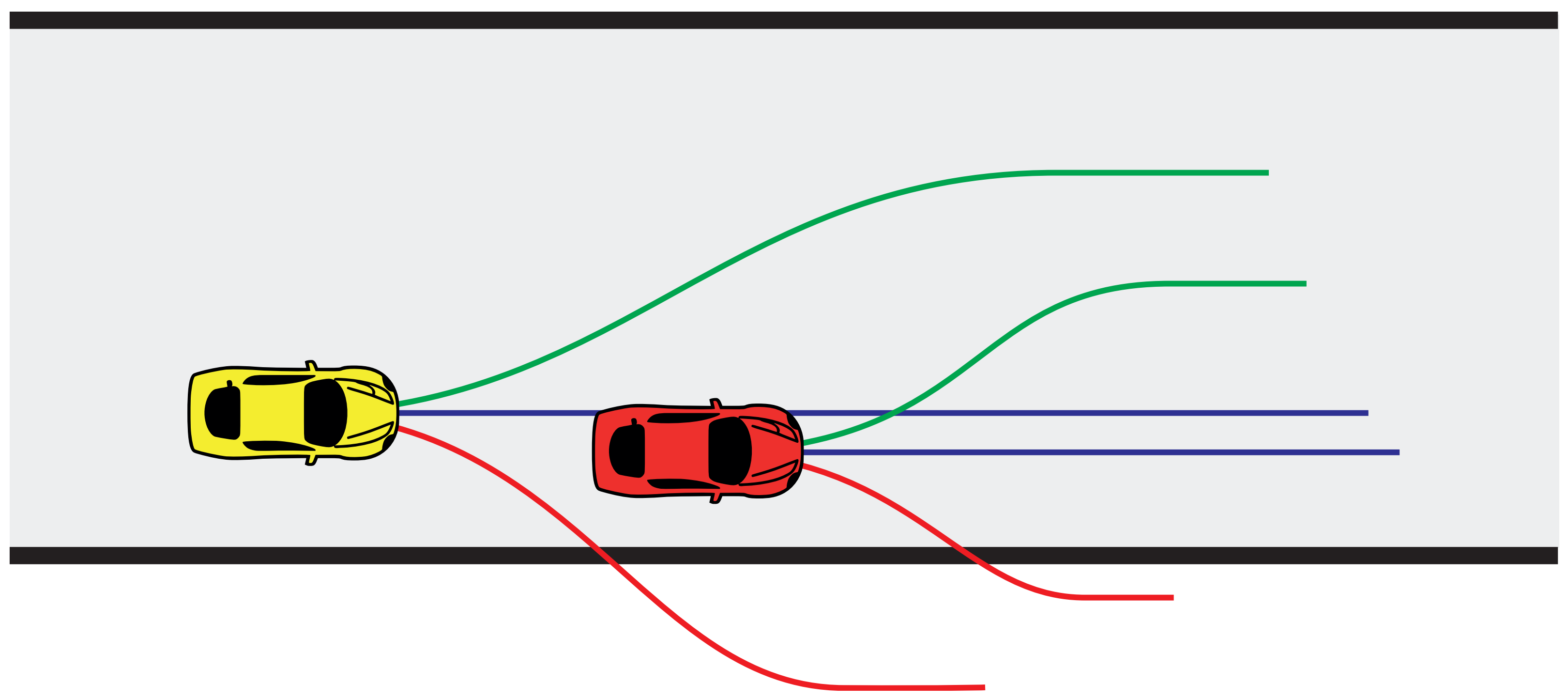
Nash Equilibria

- None of the players has a benefit from **unilaterally** changing the trajectory

$$a_{i^*,j^*} \geq a_{i,j^*} \quad \forall i \in \Gamma^1$$

$$b_{i^*,j^*} \geq b_{i^*,j} \quad \forall j \in \Gamma^2$$

Sequential and Cooperative Game



sequential game

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cooperative game

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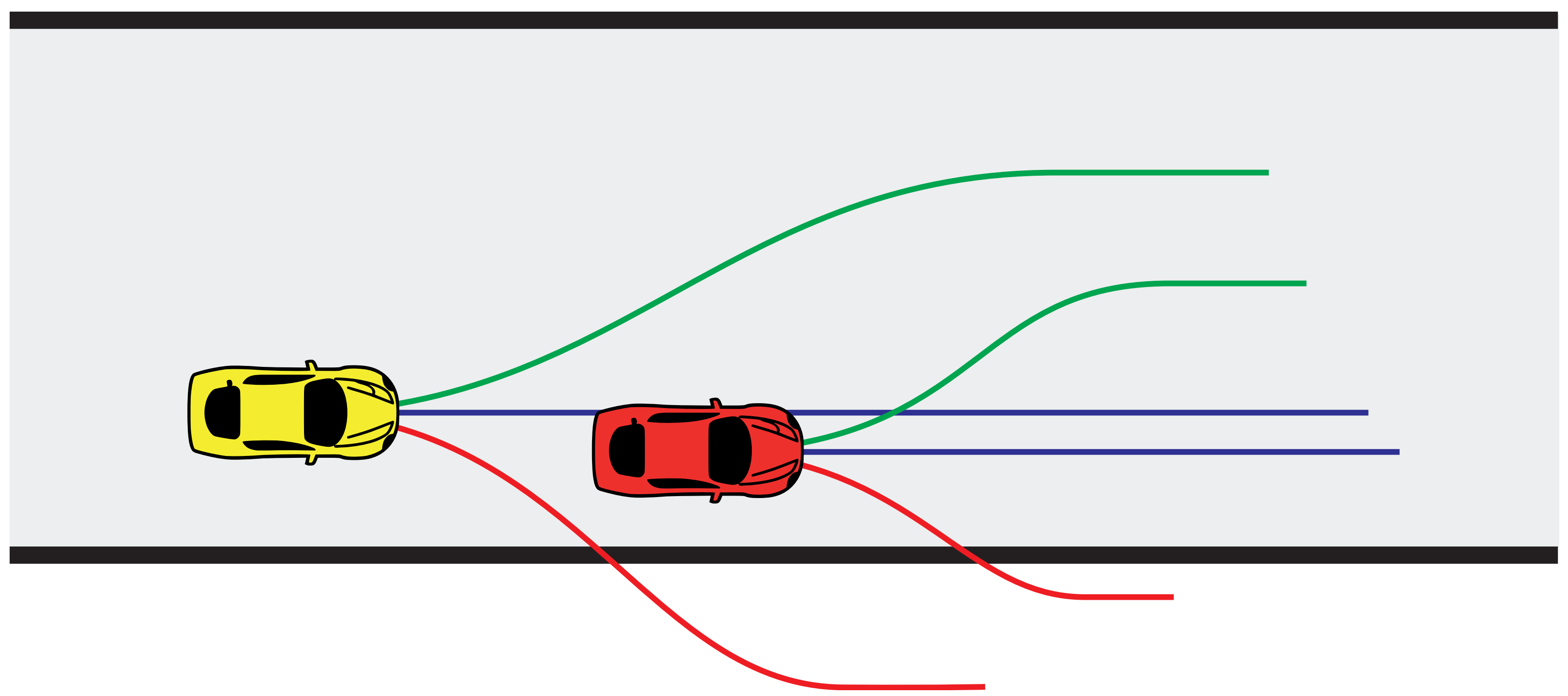

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- The sequential game can be solved by sequential maximizing

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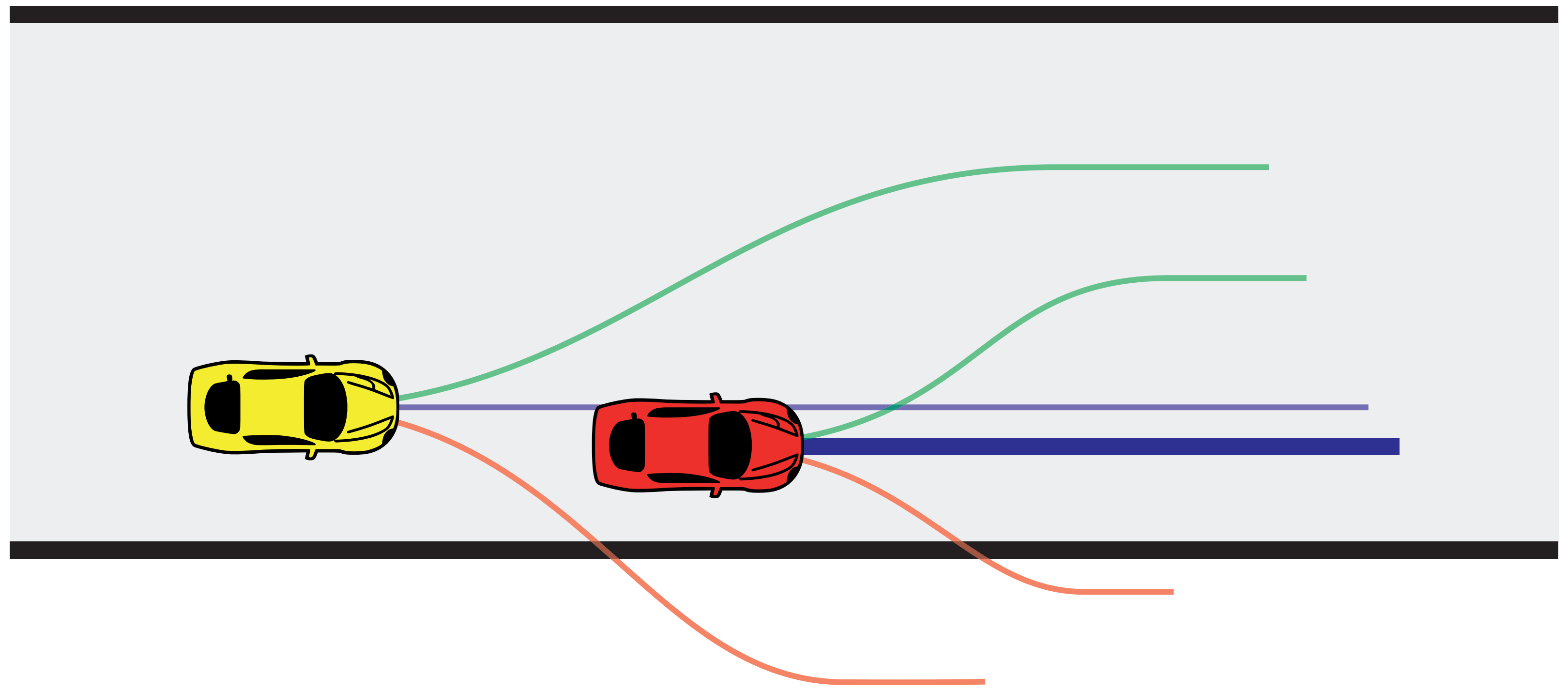

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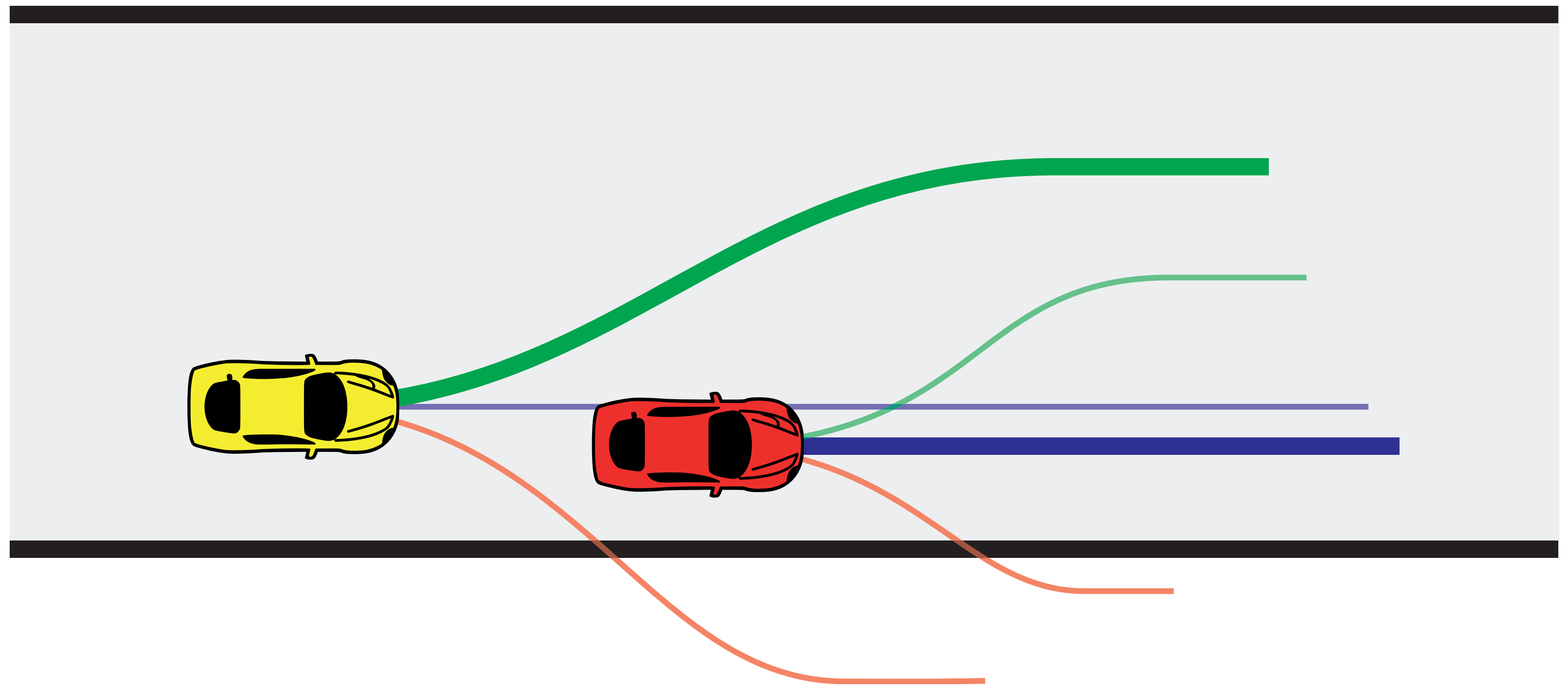

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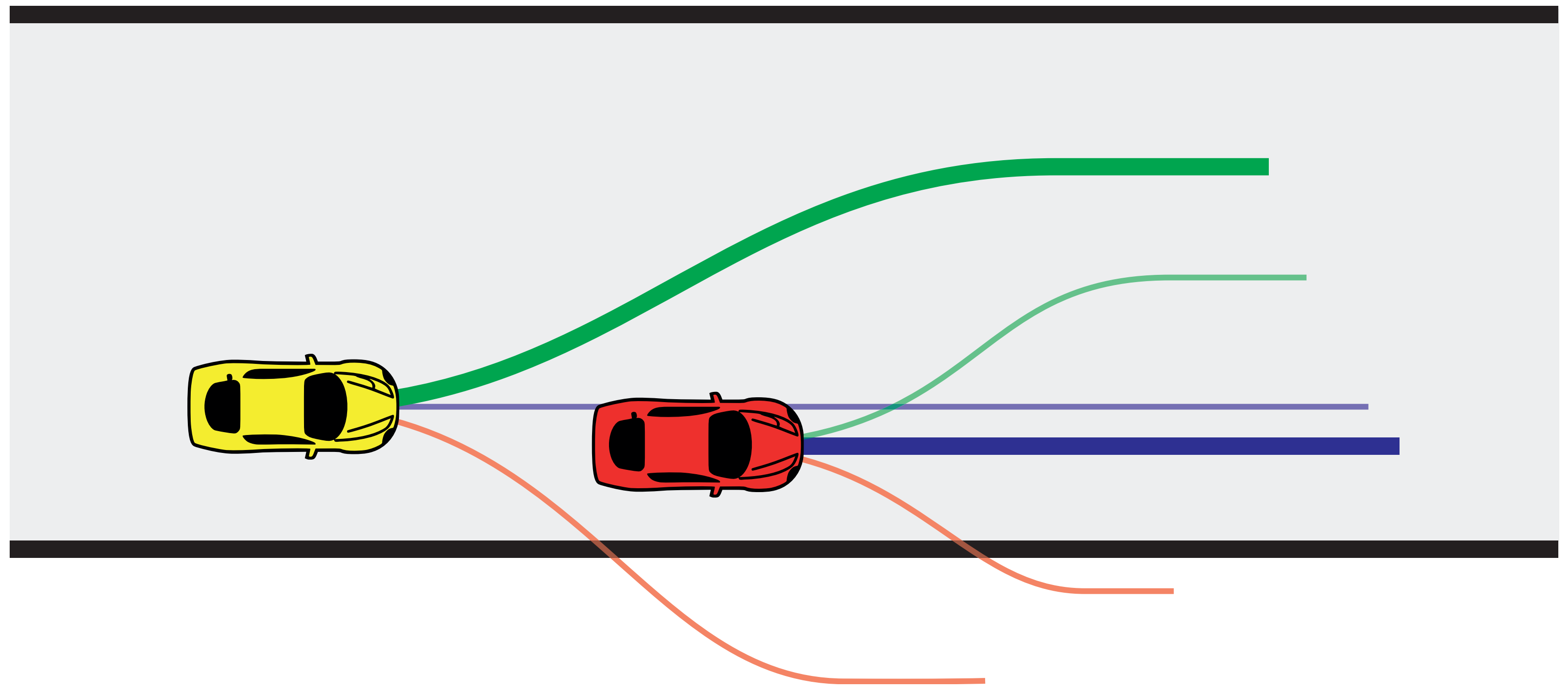

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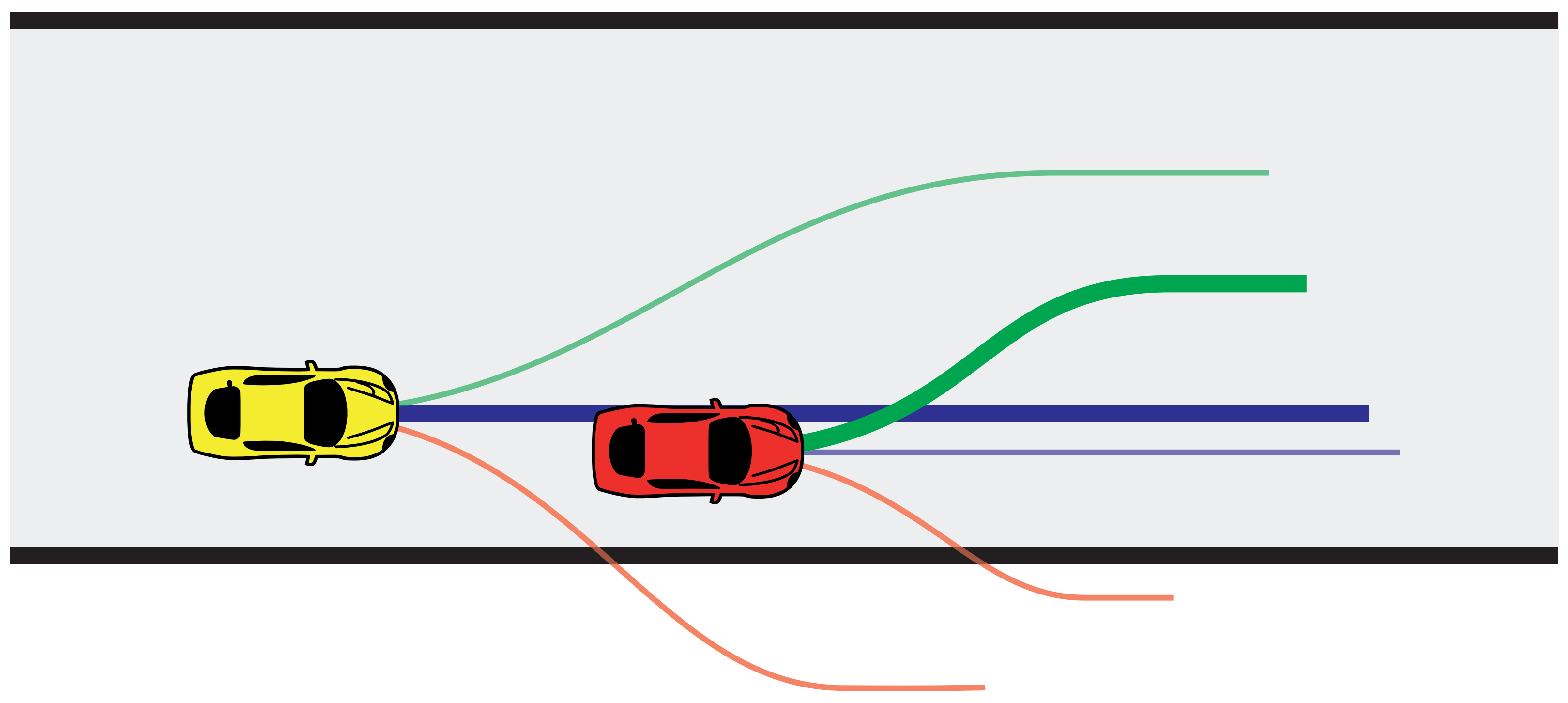

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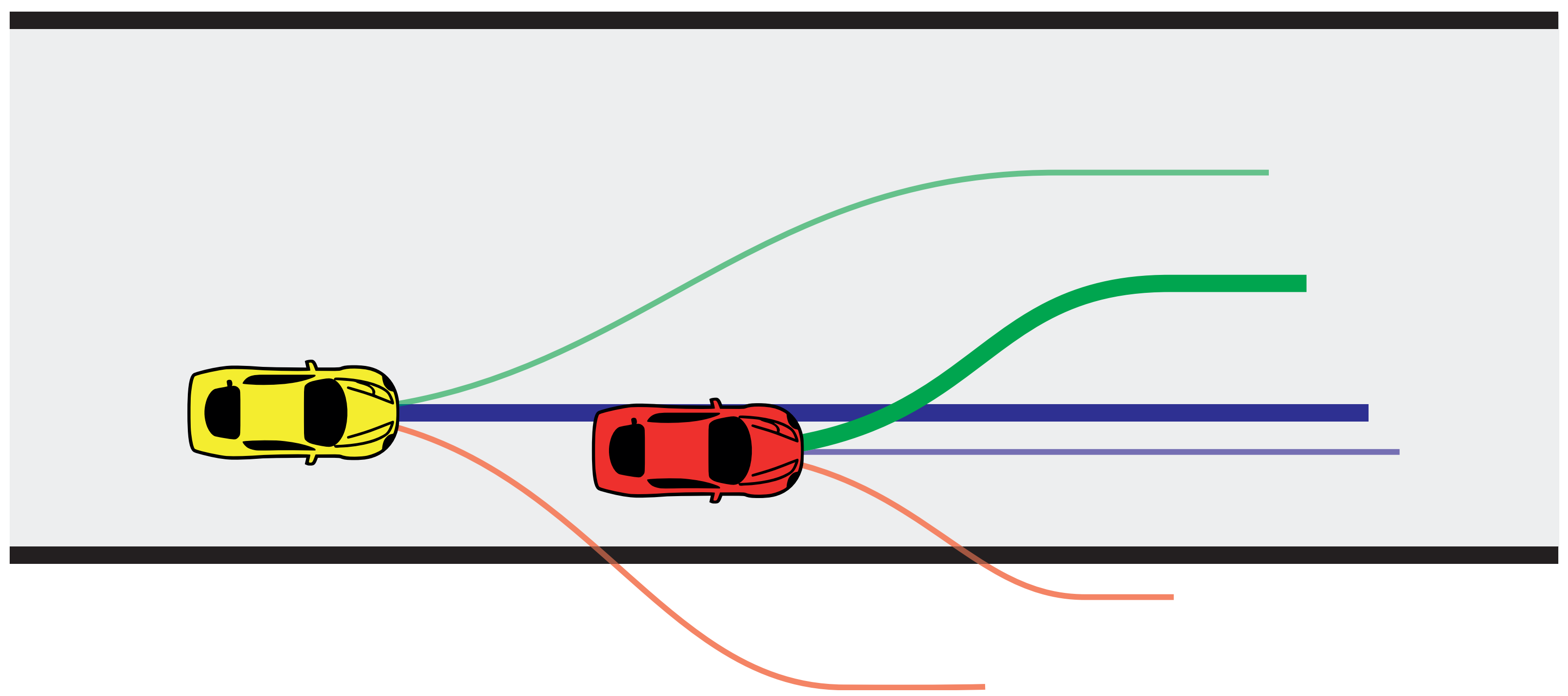
Sequential and Cooperative Game



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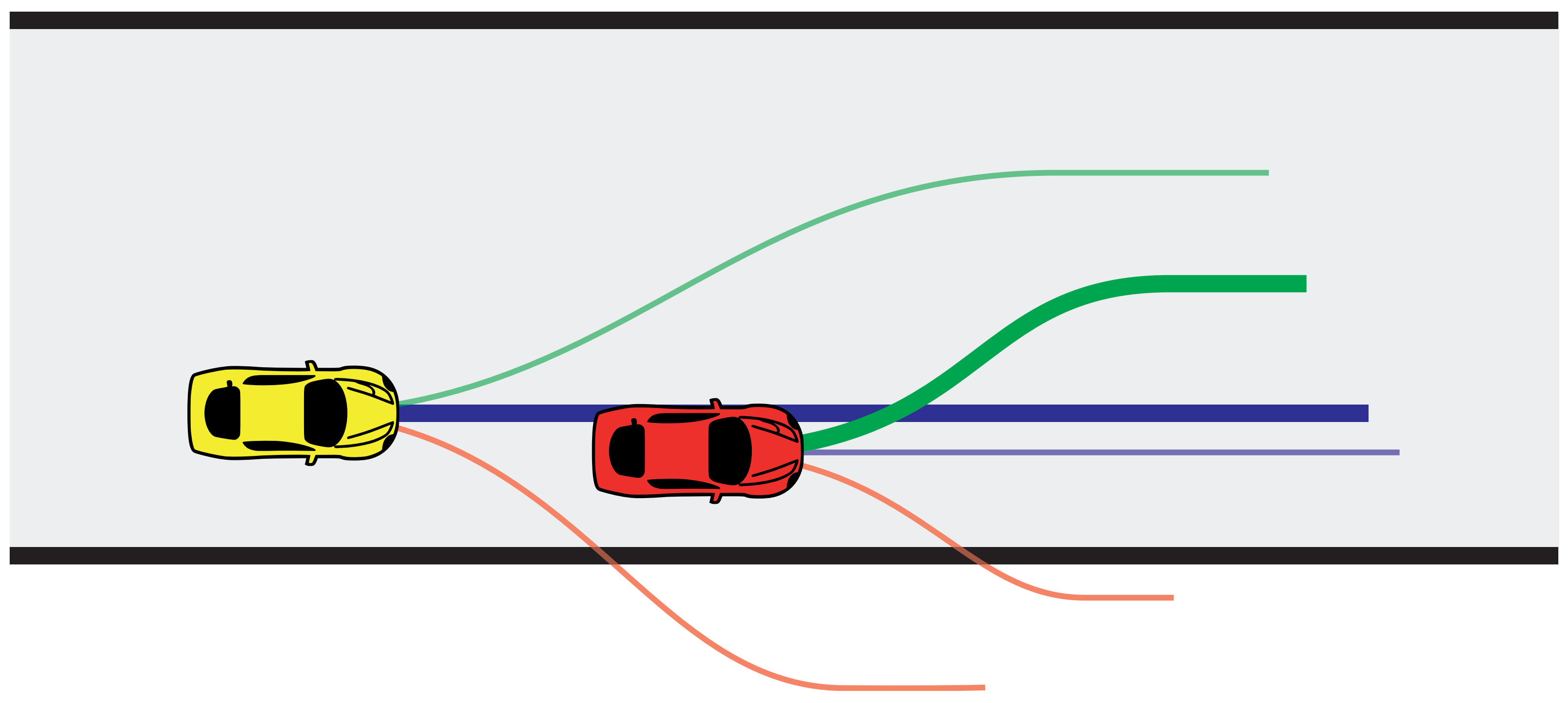
- The sequential game can be solved by sequential maximizing
- Sequential game feasible —> equilibrium of the cooperative game
  - Predicting ideal behavior of other cars and play best response is a Nash equilibrium

Sequential and Cooperative Game



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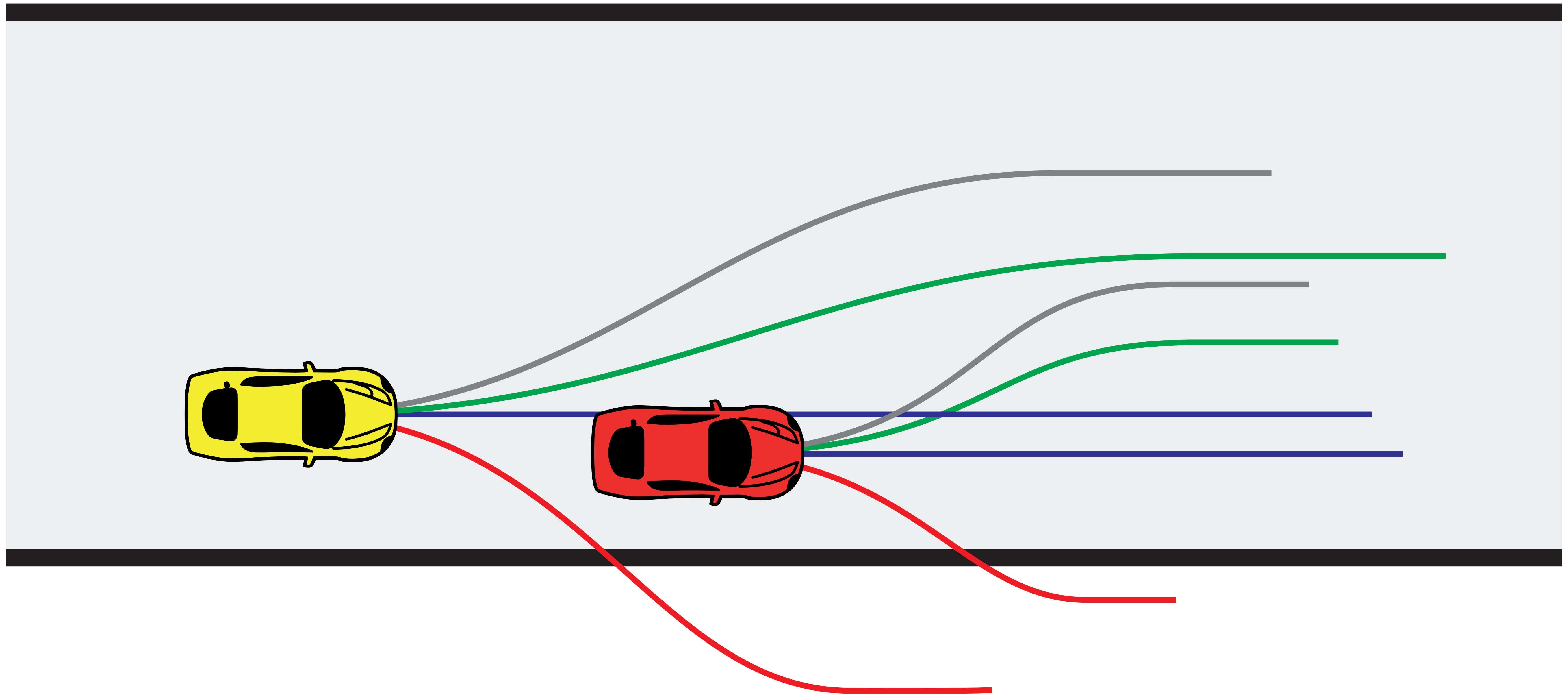
- Sequential game feasible —> equilibrium of the cooperative game
  - Predicting ideal behavior of other cars and play best response is a Nash equilibrium
- Cooperative game is feasible if there exists a feasible trajectory pair

Blocking Trajectories



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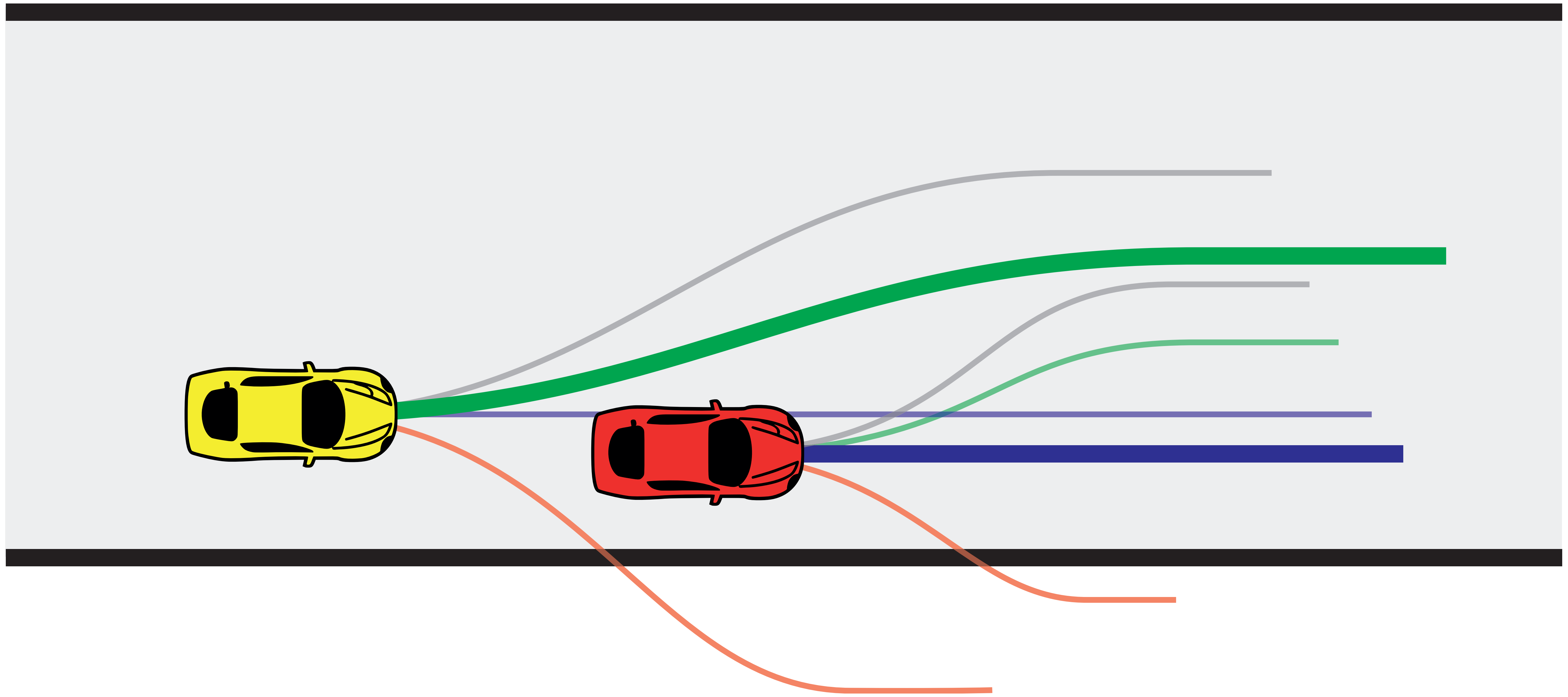

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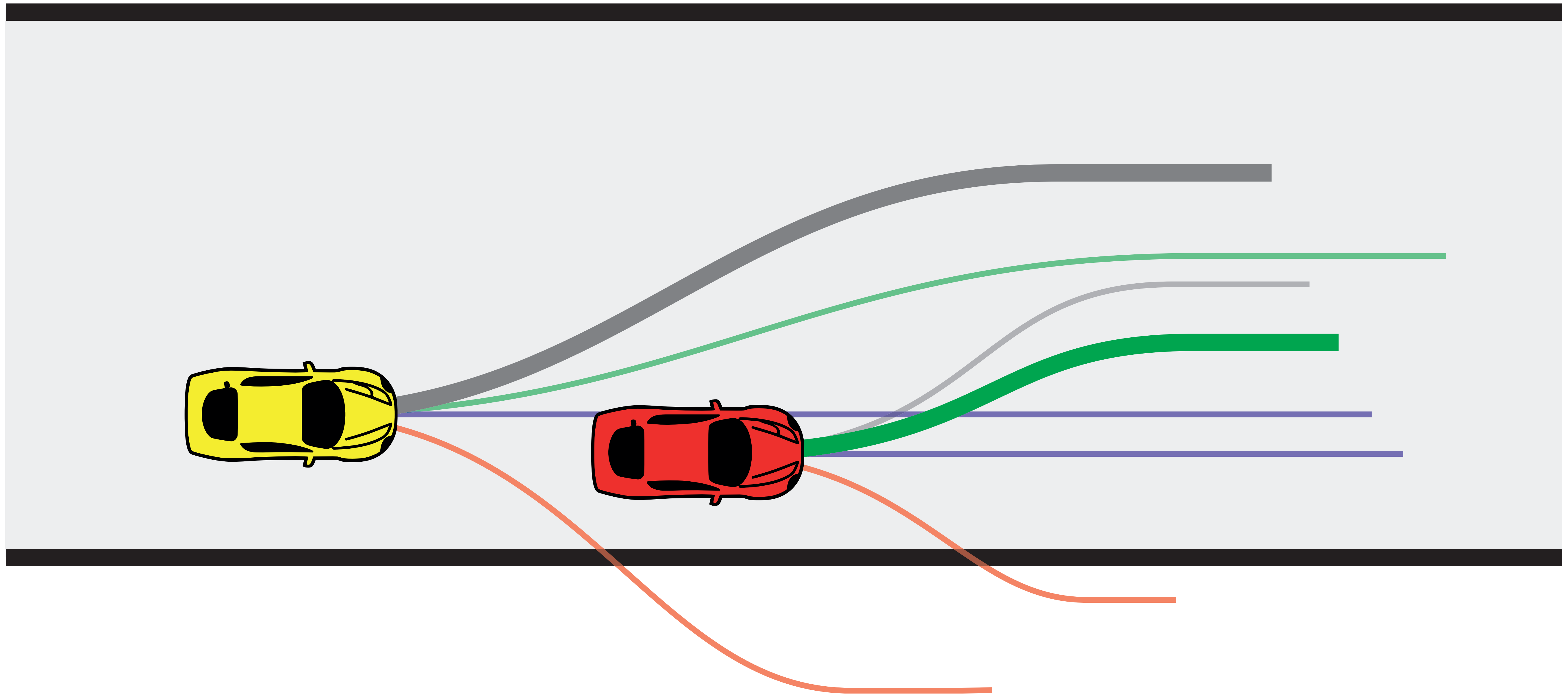

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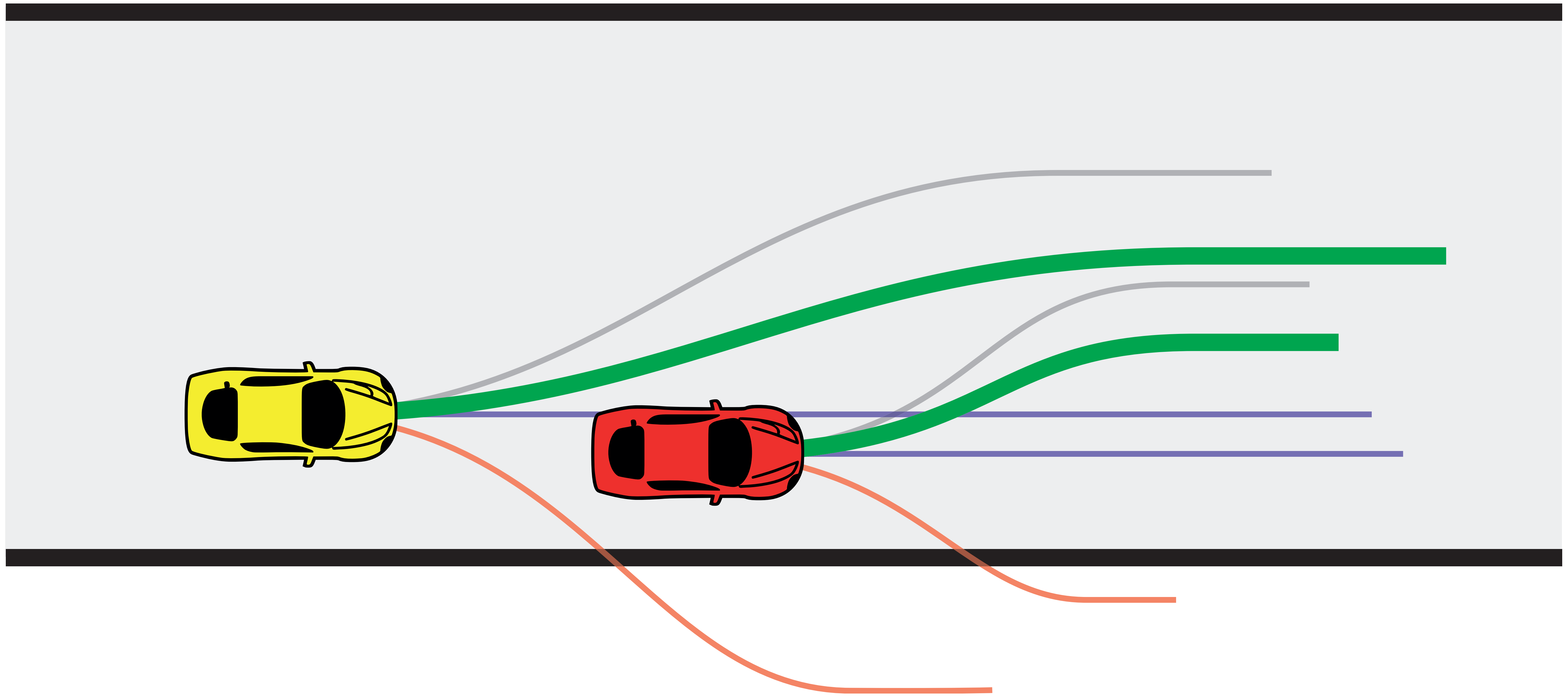

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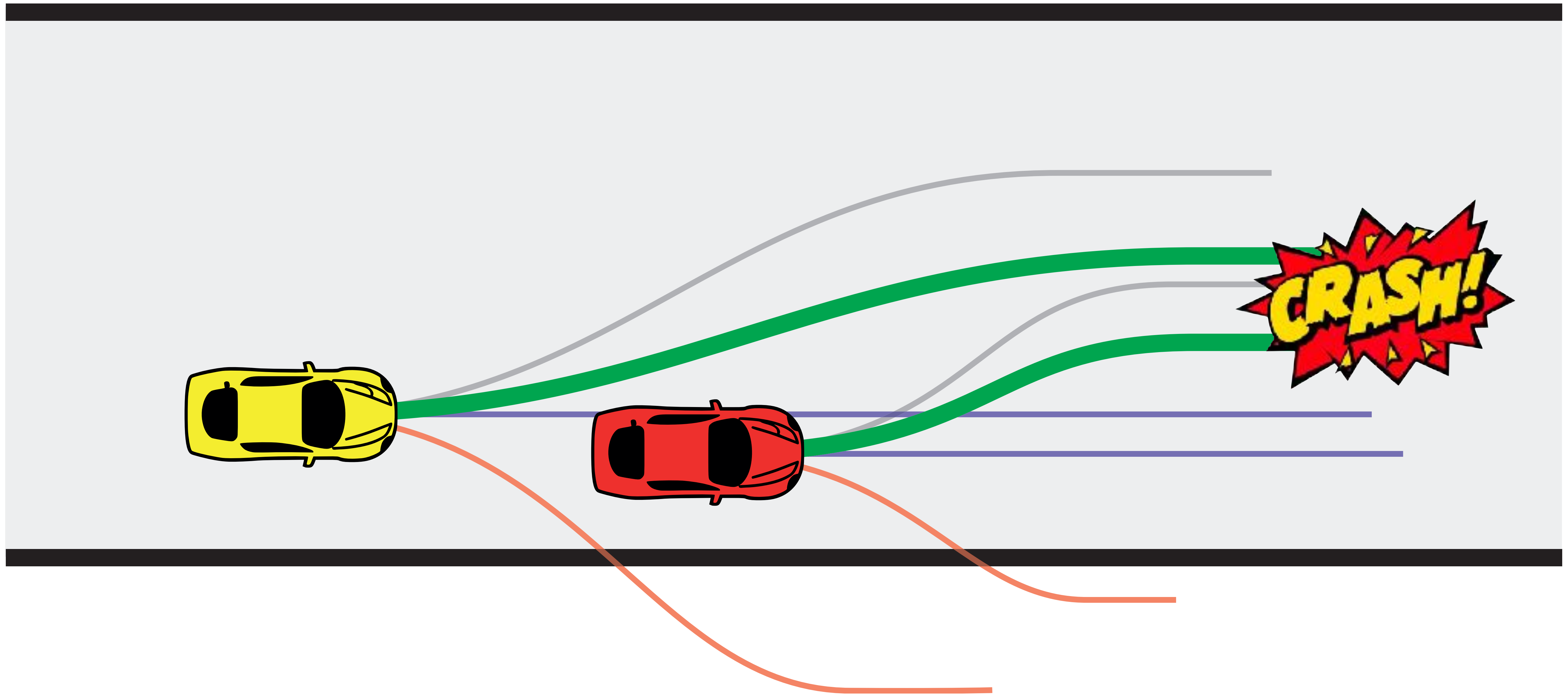

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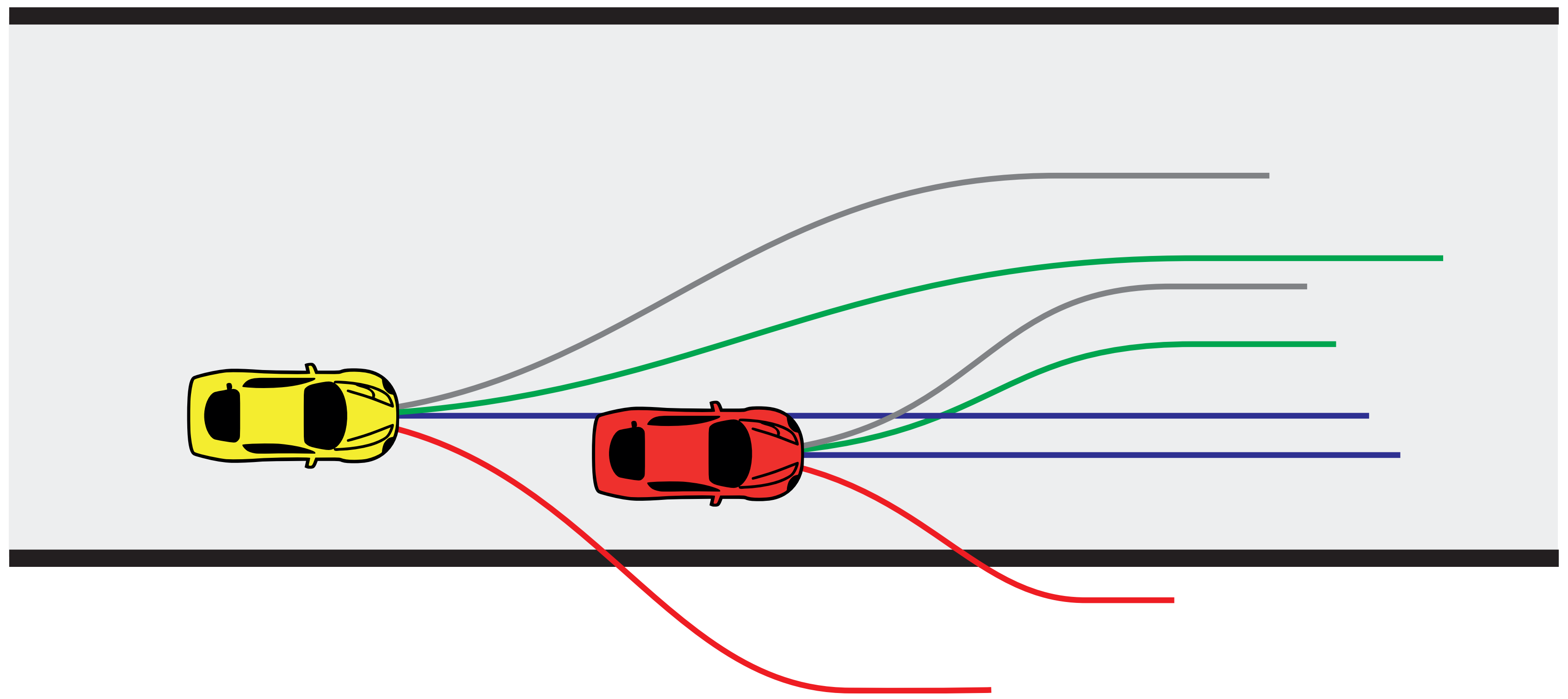

$$A = \begin{bmatrix} 0.83+0.5 & -1 & 0.83 & 0.83+0.5 \\ 0.85+0.5 & -1 & -1 & 0.85+0.5 \\ 0.88+0.5 & 0.88 & -1 & 0.88+0.5 \\ -10 & -10 & -10 & -10 \end{bmatrix}$$

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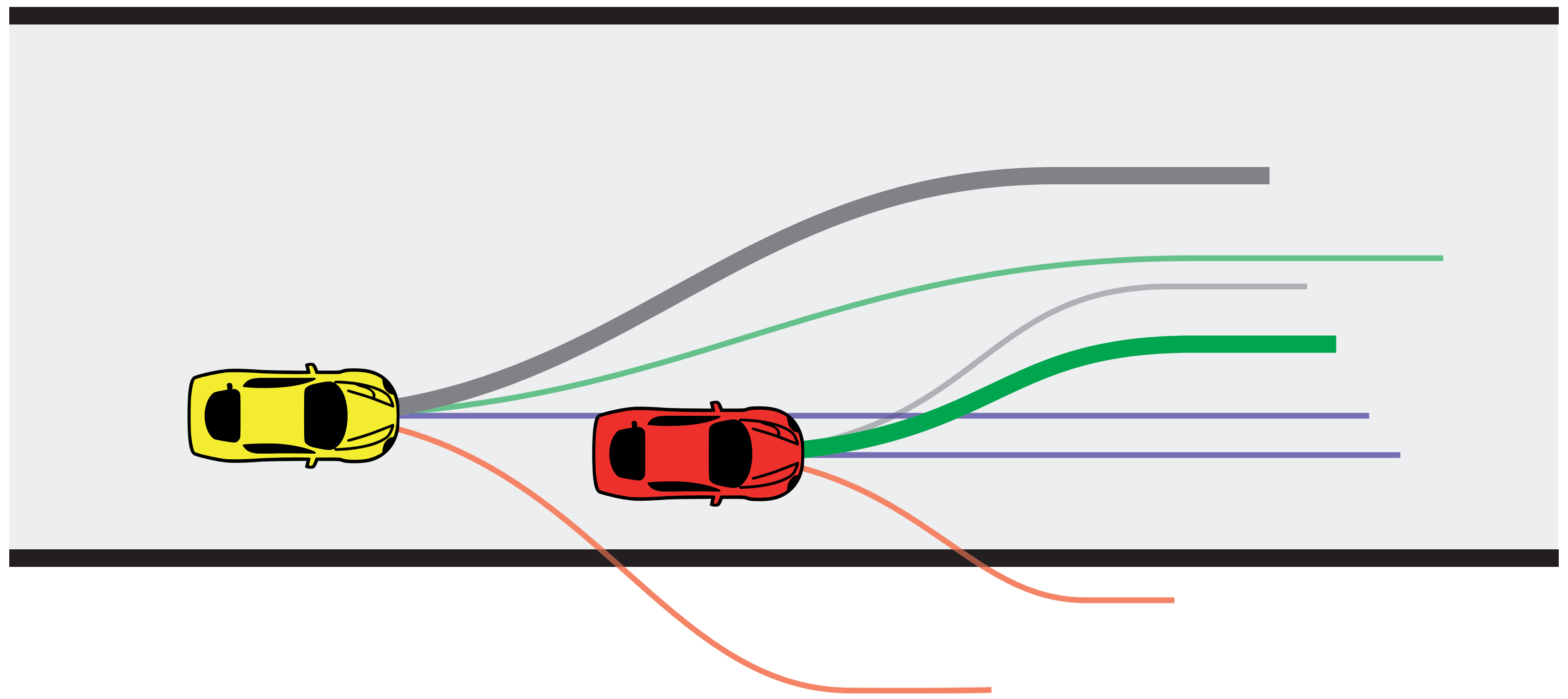

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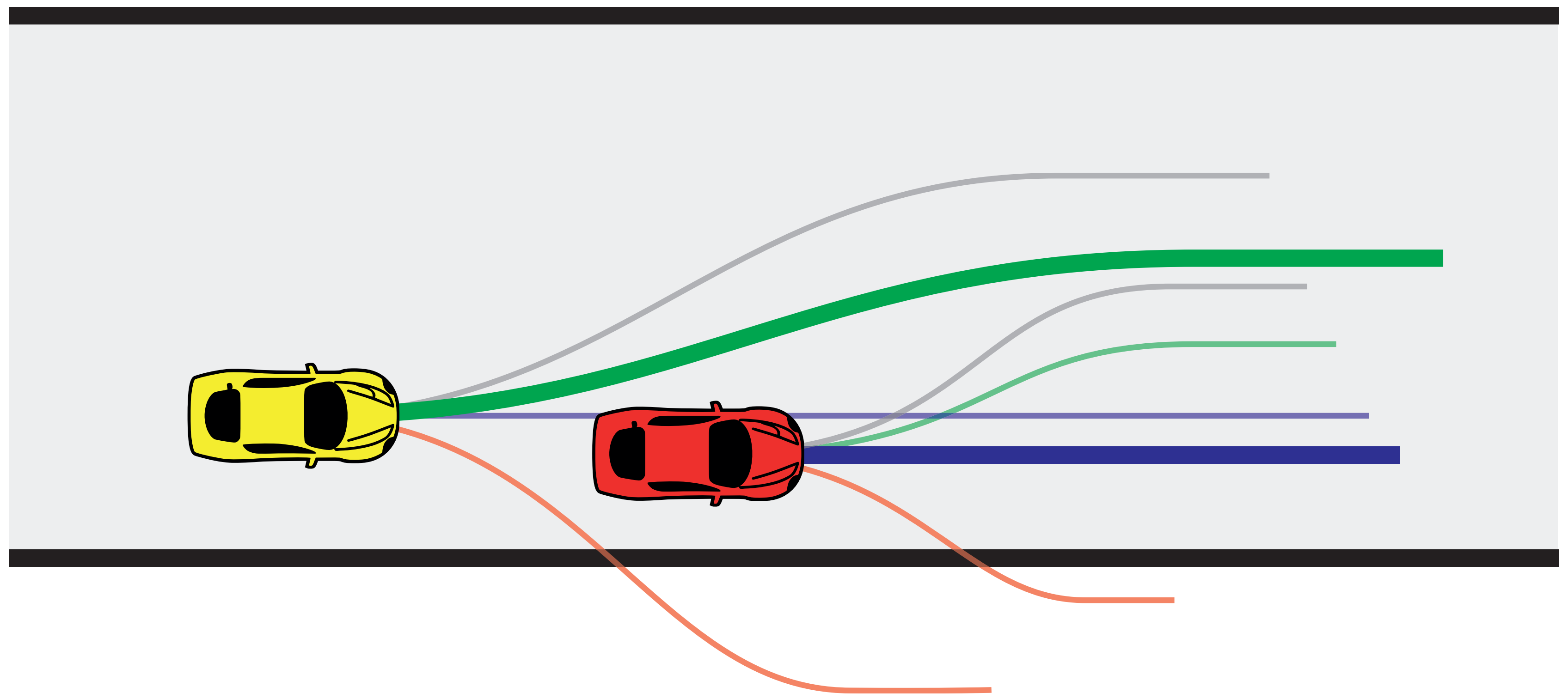
- If there exists a blocking trajectory and the **staying ahead reward** is big enough, the Stackelberg equilibrium is a blocking trajectory pair

Blocking Trajectories



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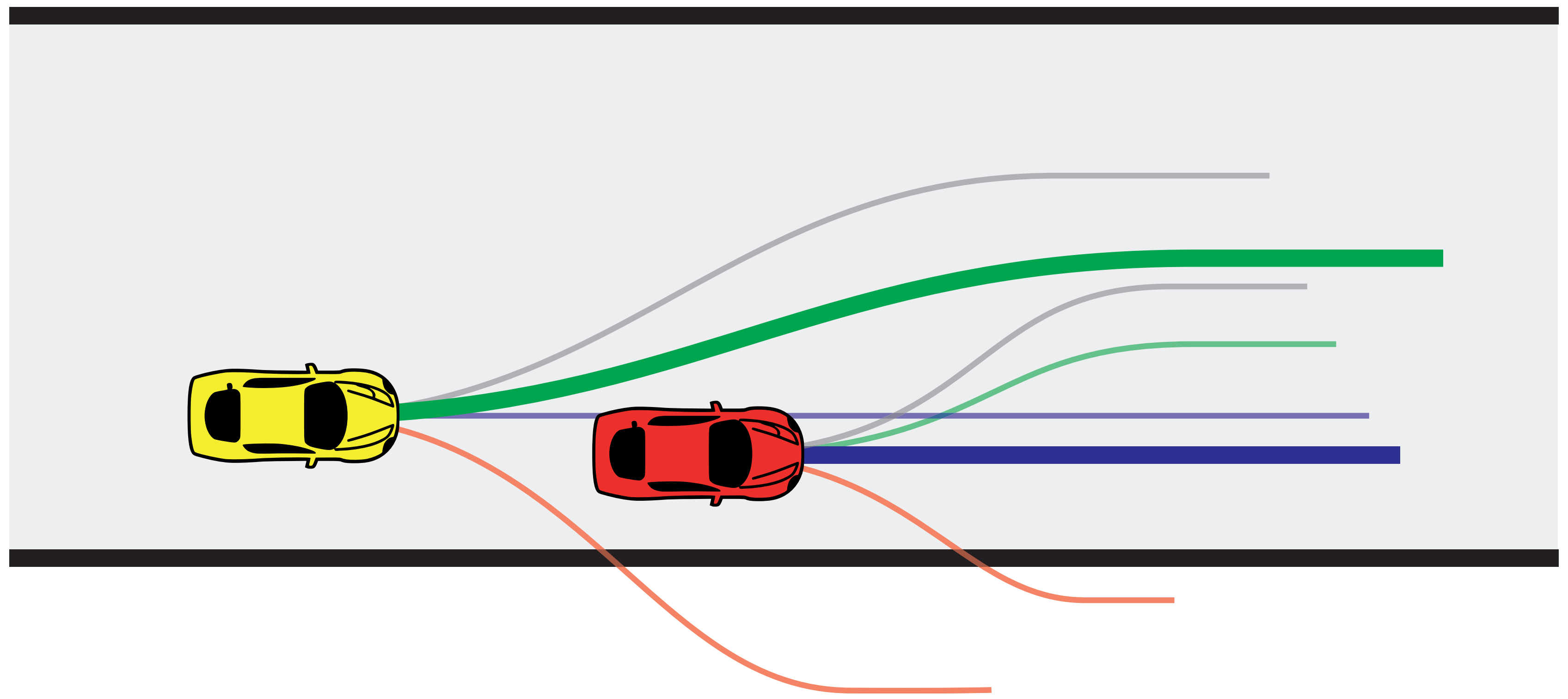
Blocking Trajectories



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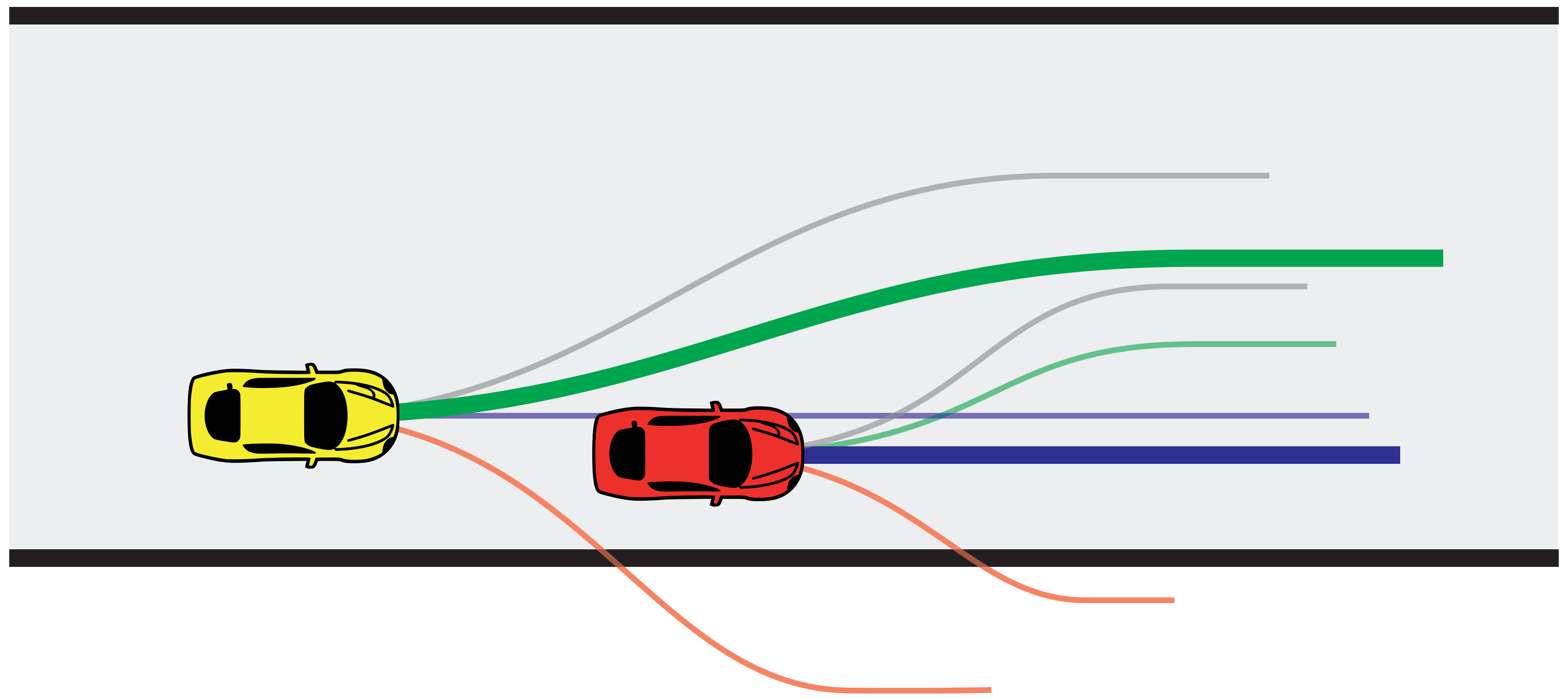
Stackelberg equilibrium seems best for all games

Blocking Trajectories



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Stackelberg equilibrium seems best for all games

What is the resulting behavior of these games?

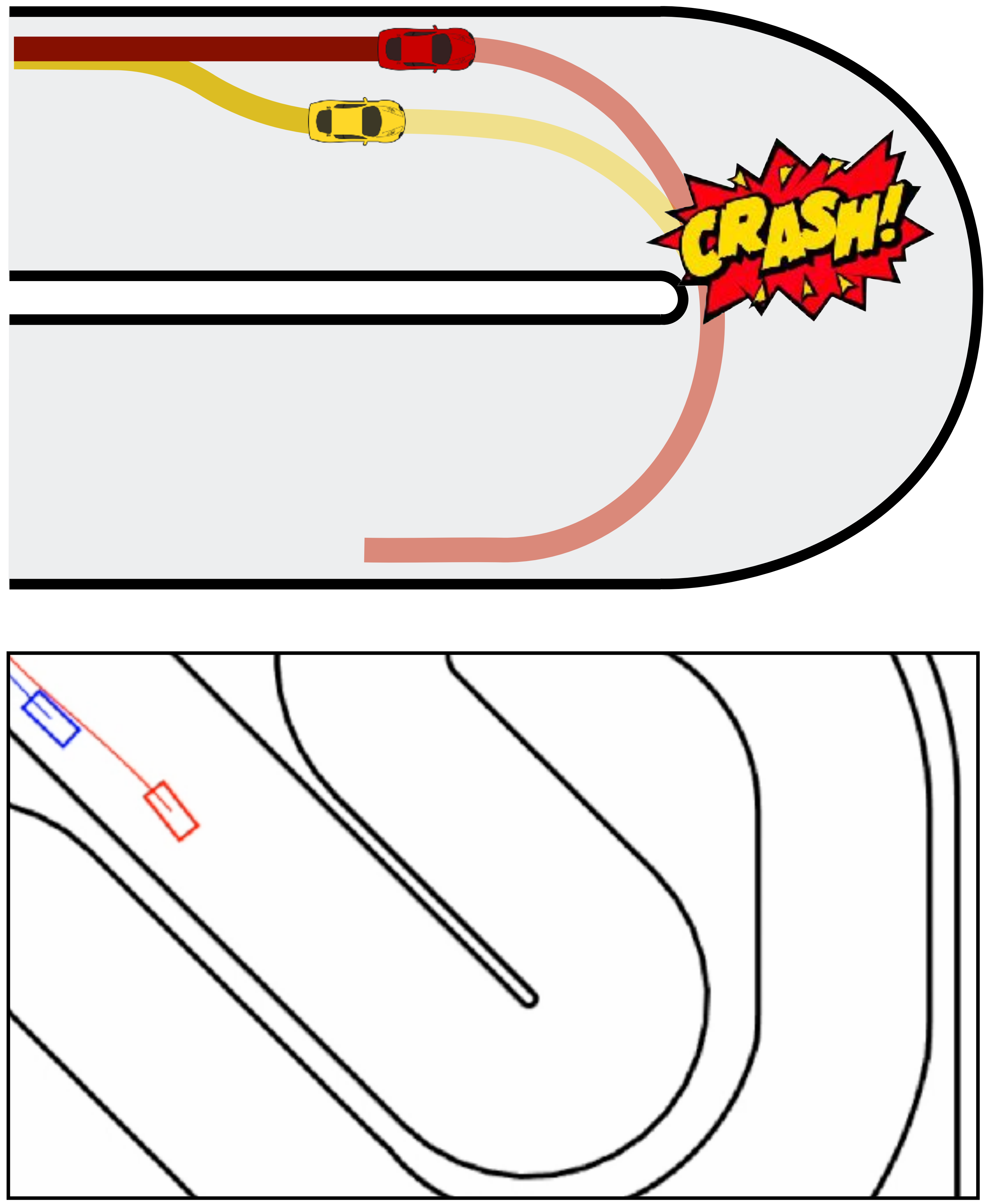
Simulation

- Play game in a receding horizon fashion
  - Solve game + MPC - apply first input - repeat
- Trajectory pruning based on viability and discriminating kernel
  - Viab —> aggressive driver / Disc —> cautious driver
- 500 different initial conditions, each run 4.5 laps
  - Both cars start close to each other

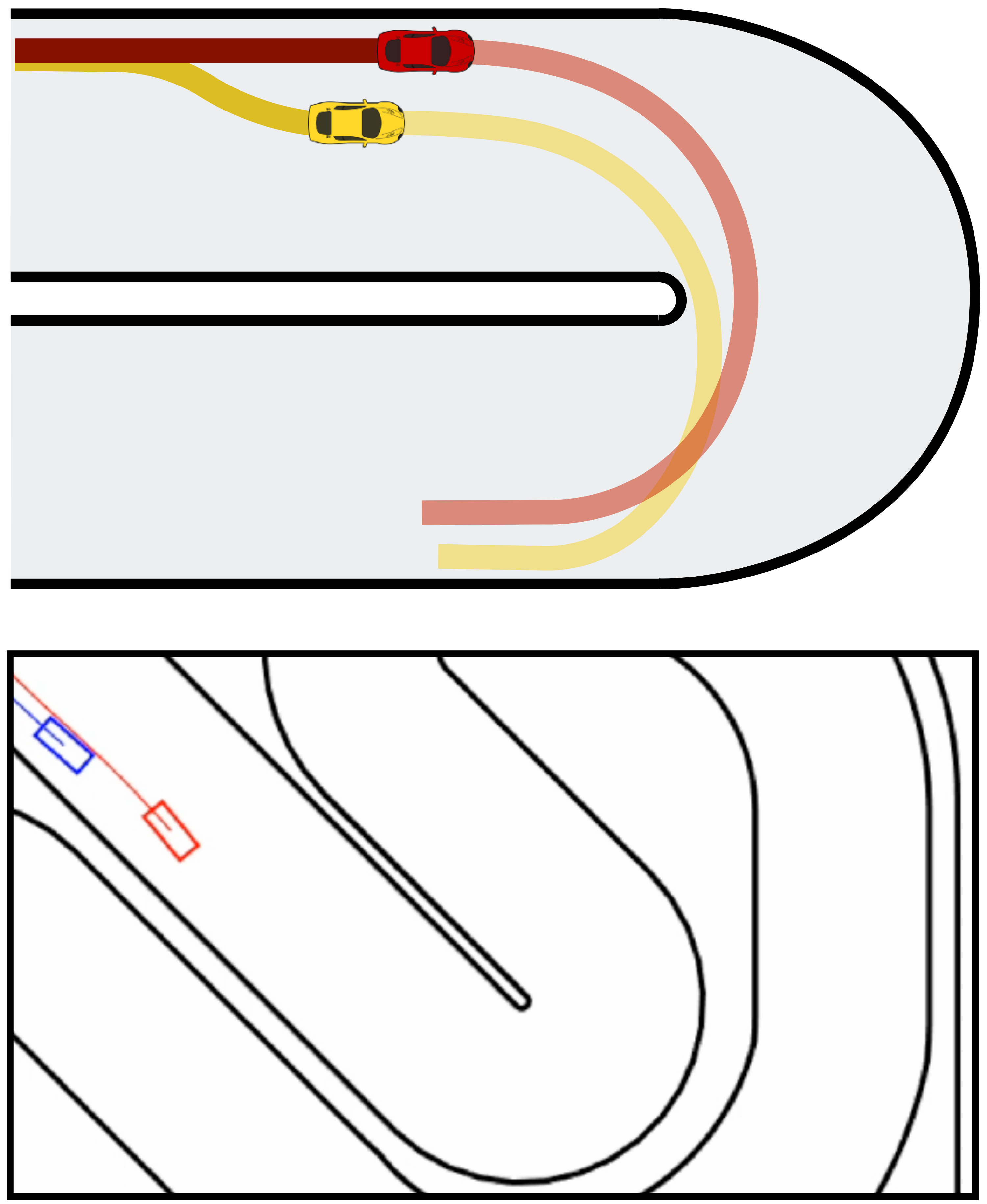
	sequential game	cooperative game	blocking game
# of overtaking maneuvers	113	857	414
colliding time steps per lap	2.4	2.0	2.3

Simulation

sequential game

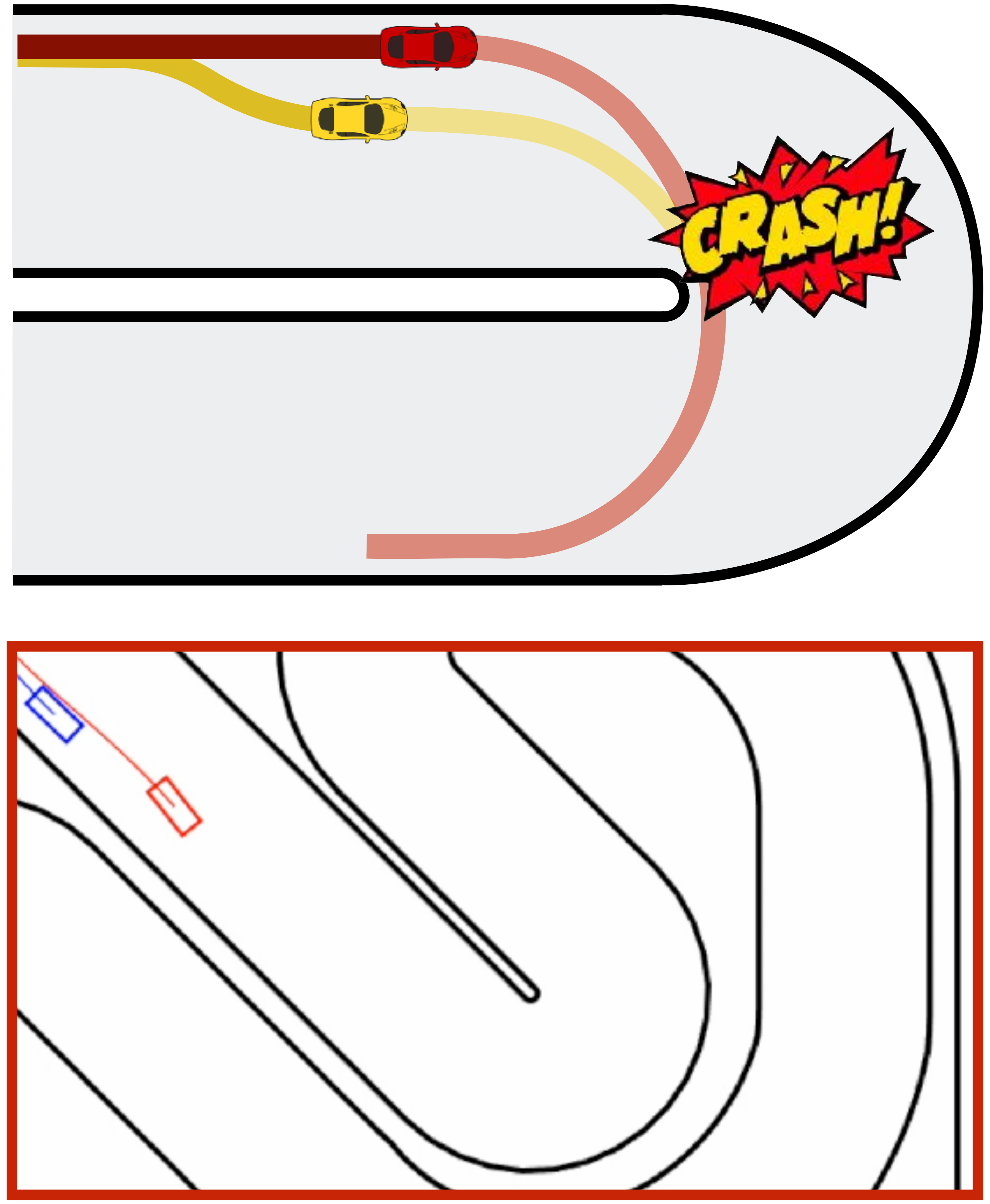


cooperative game



Simulation

sequential game



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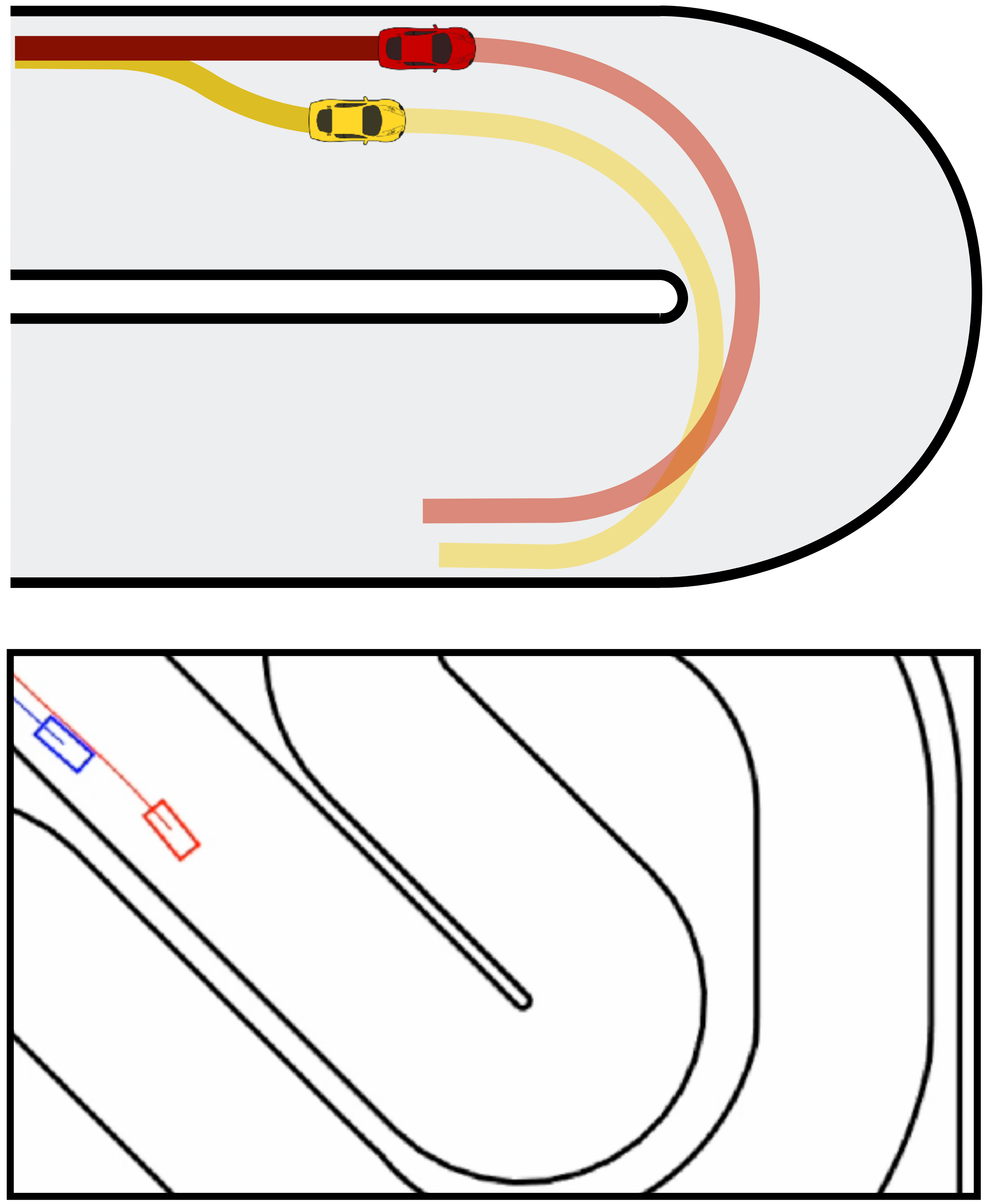

Simulation

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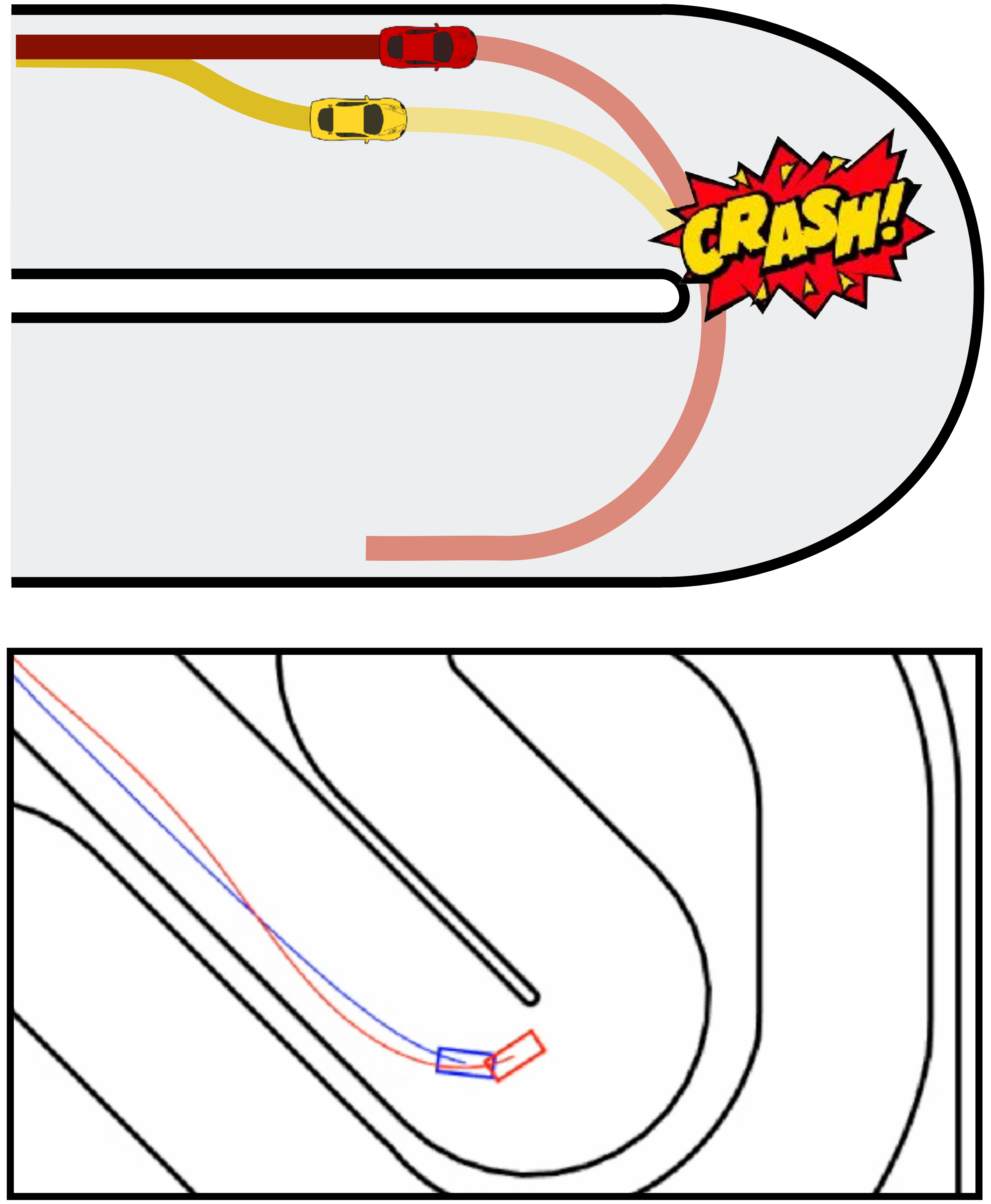
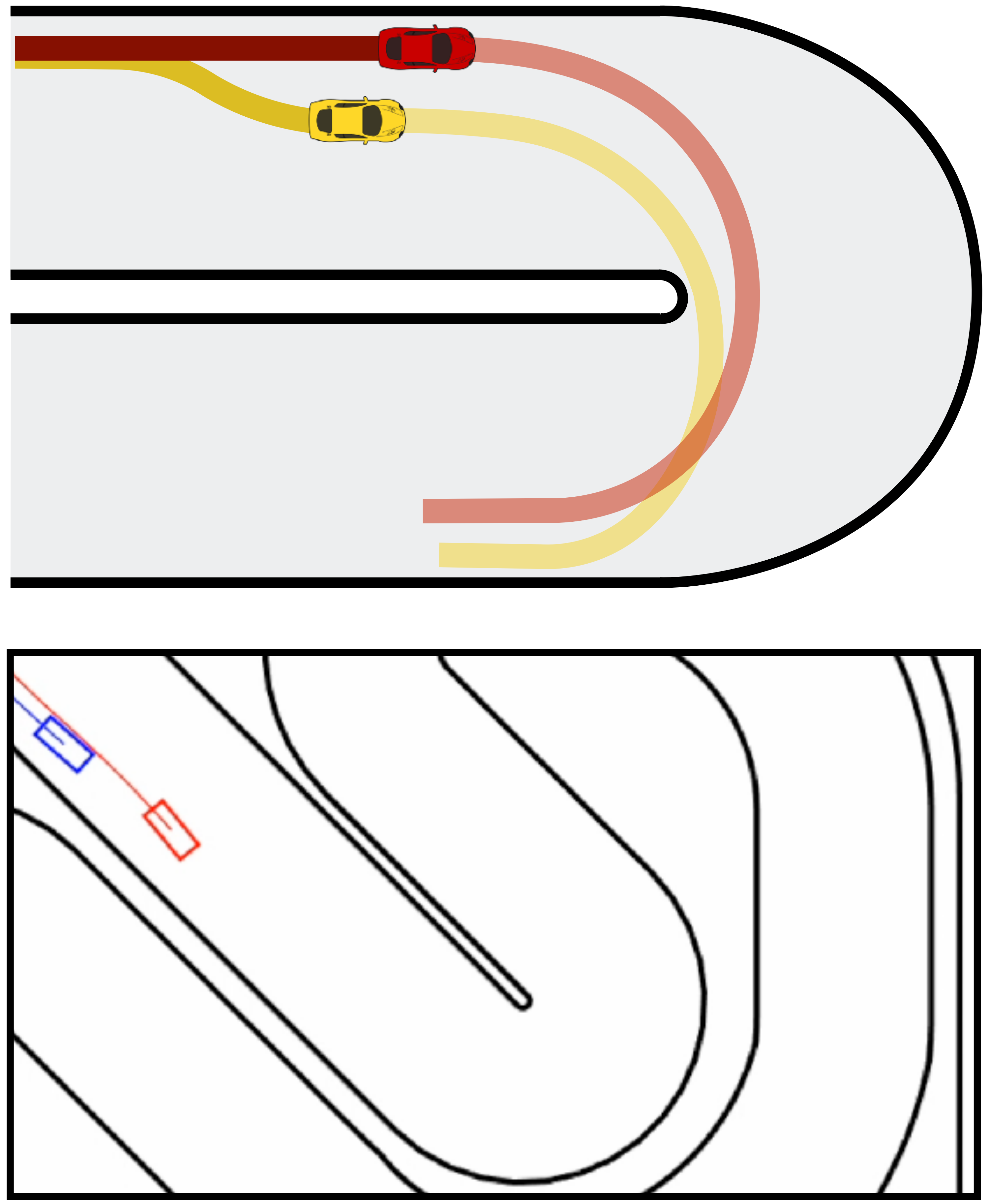
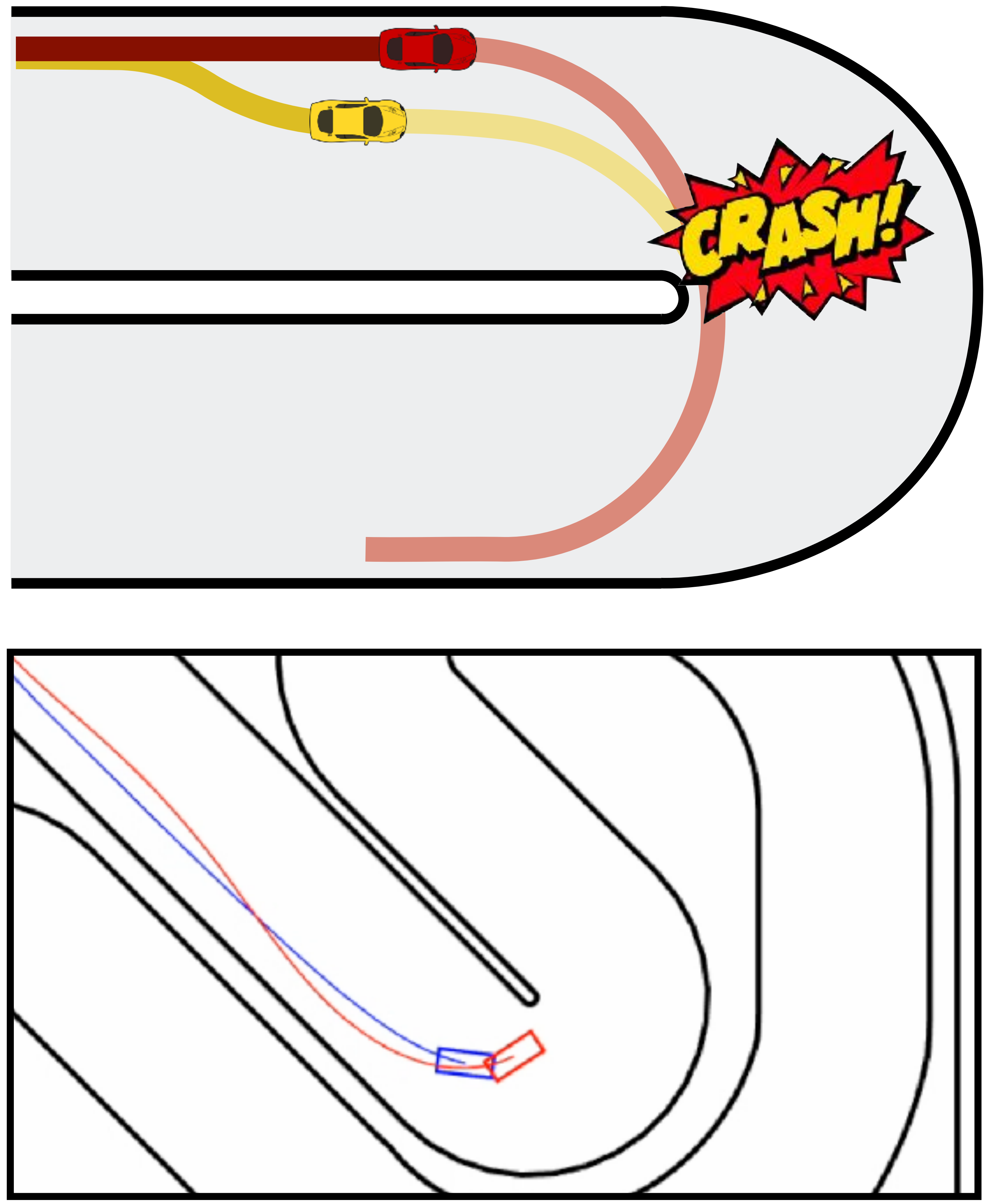


cooperative game

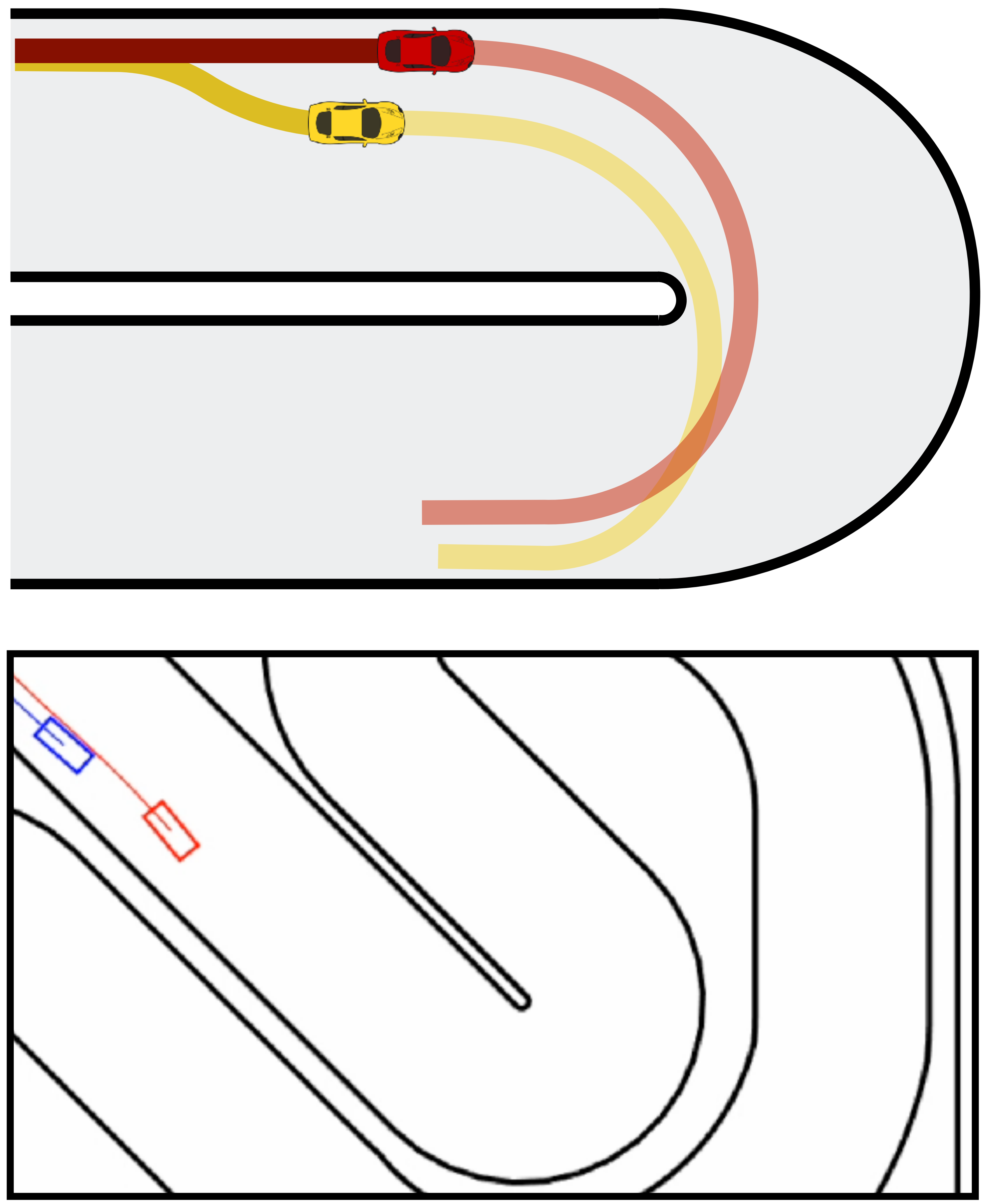


Simulation

sequential game

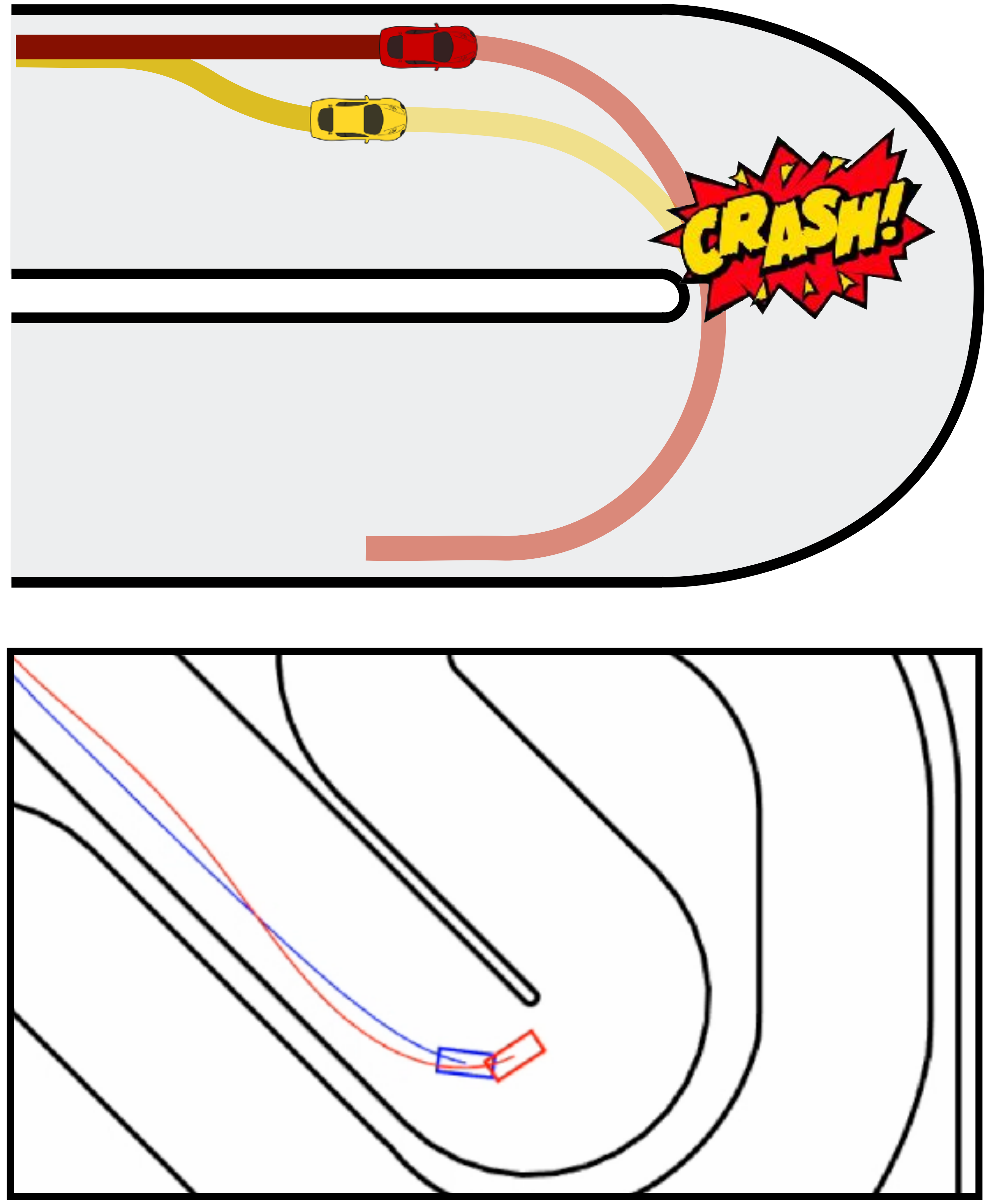


cooperative game

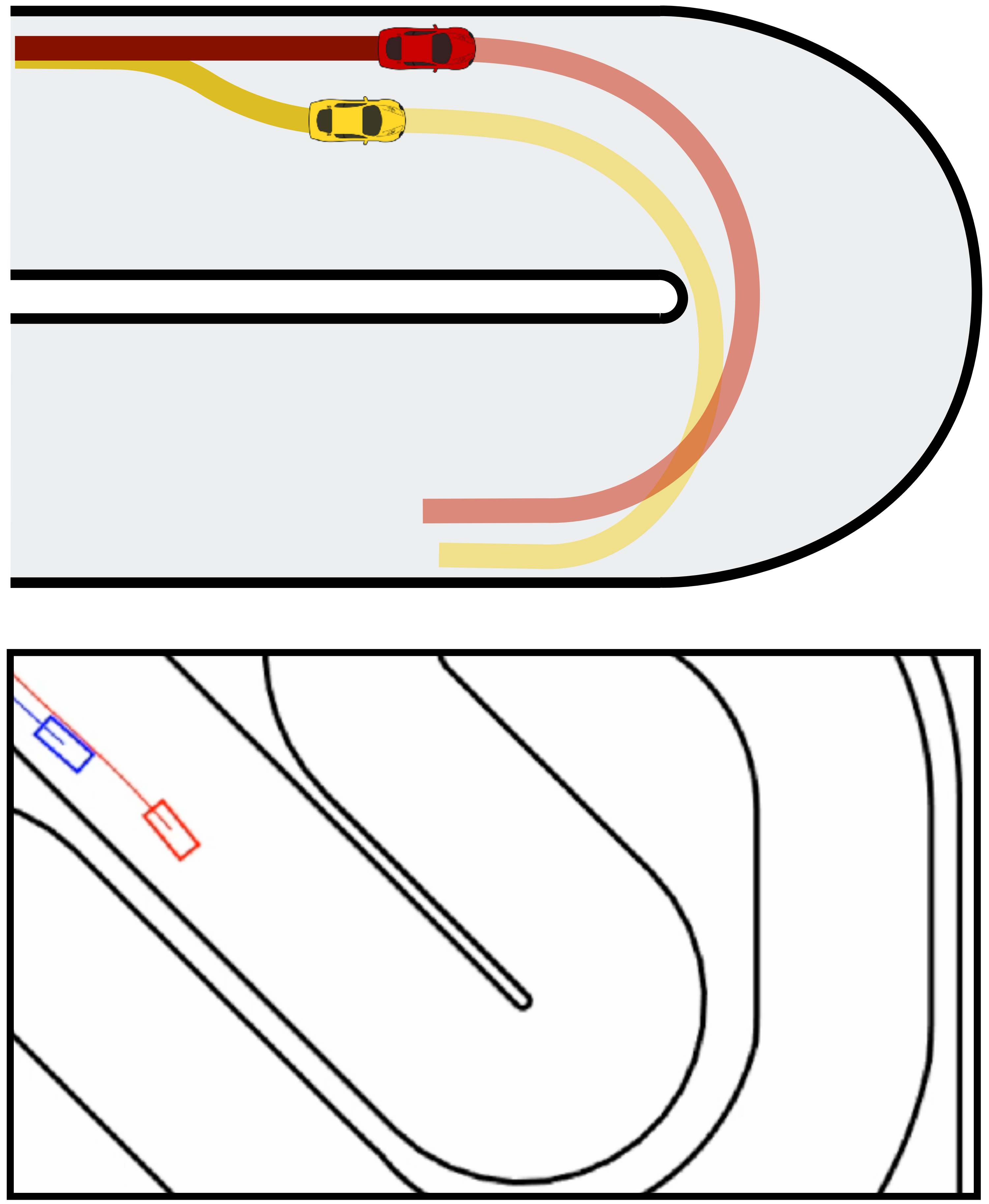


Simulation

sequential game

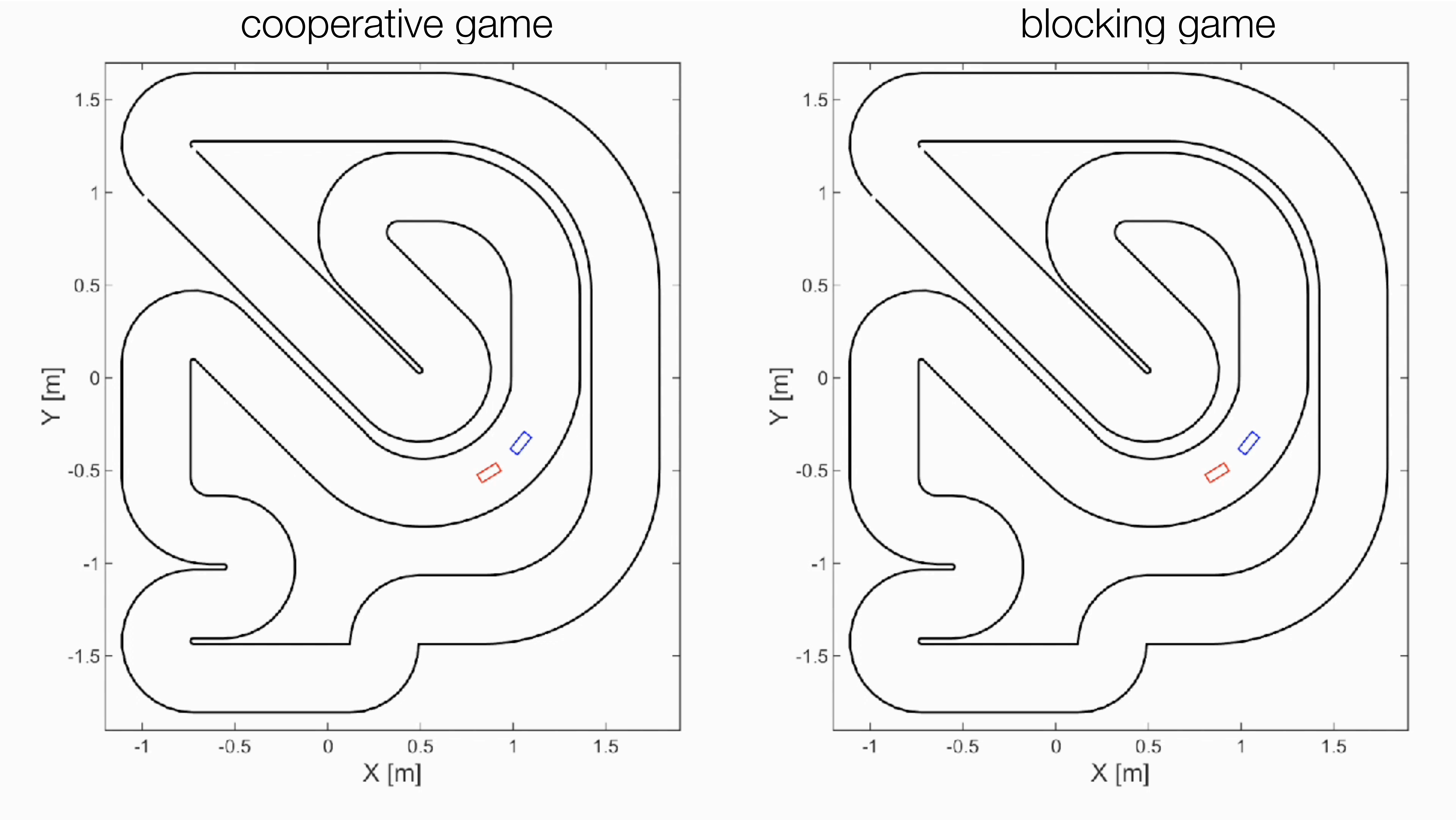


cooperative game



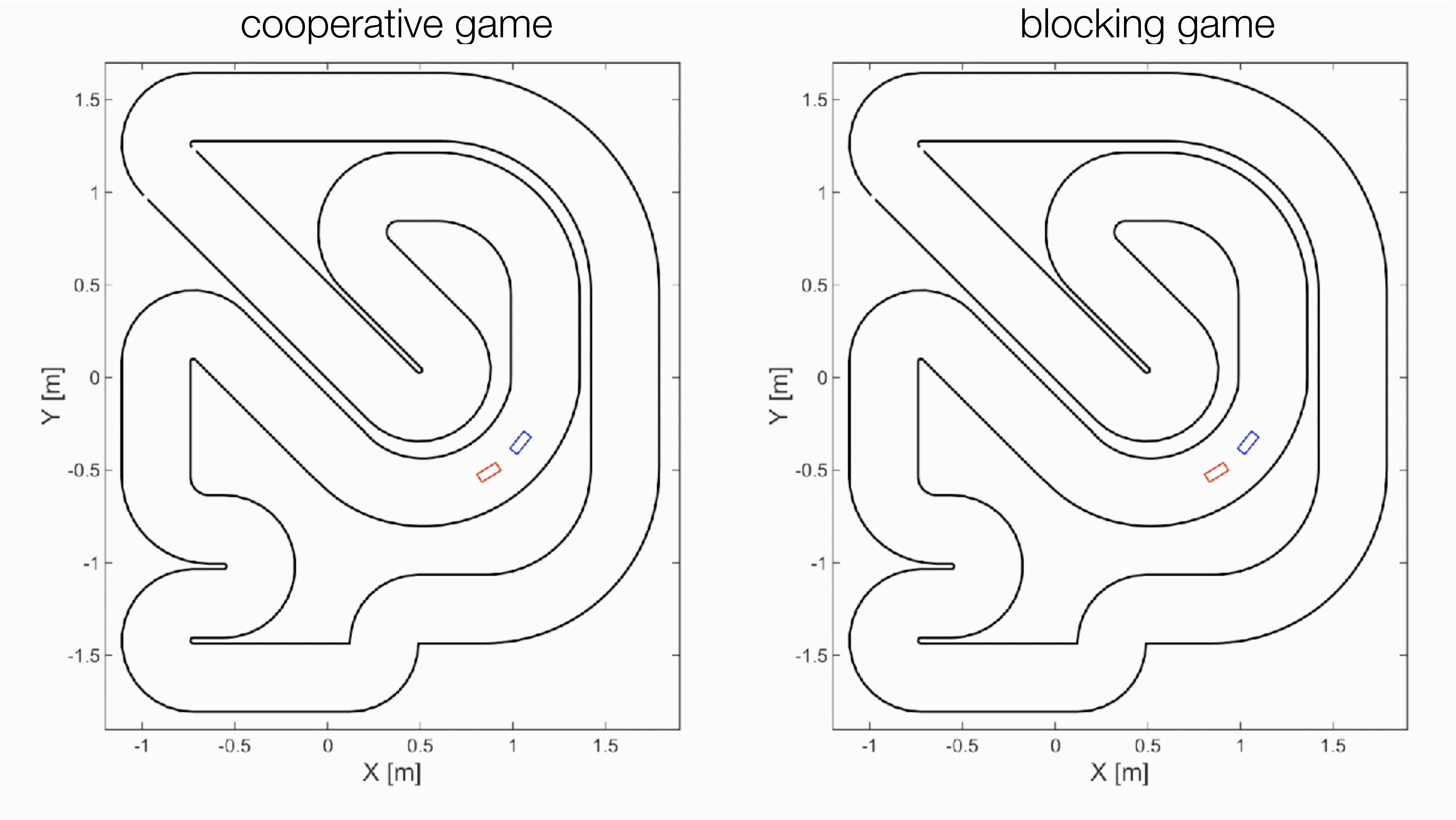
How do the cars drive?

Simulation



Viab -> aggressive driver   Disc -> cautious driver

Simulation



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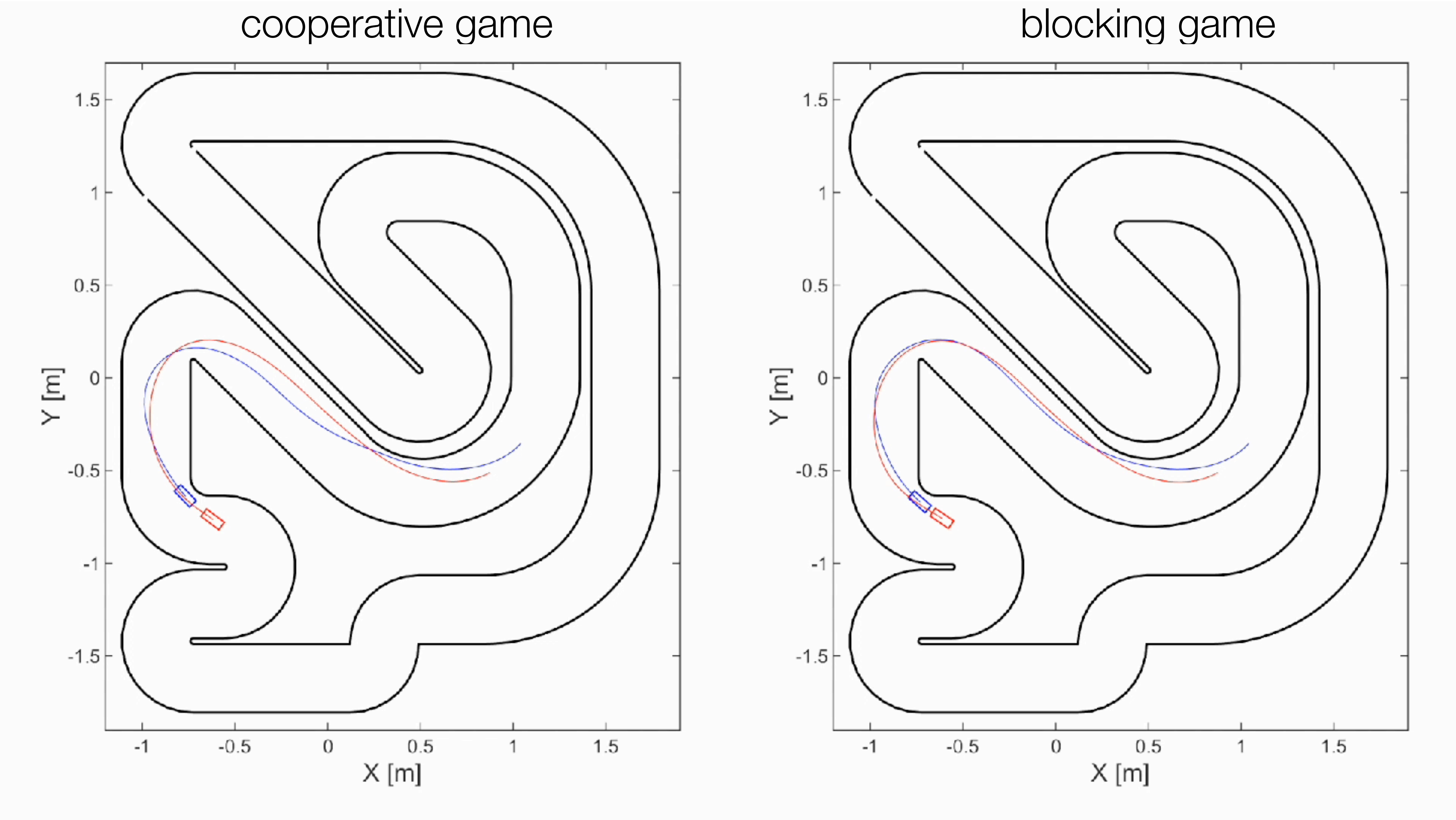

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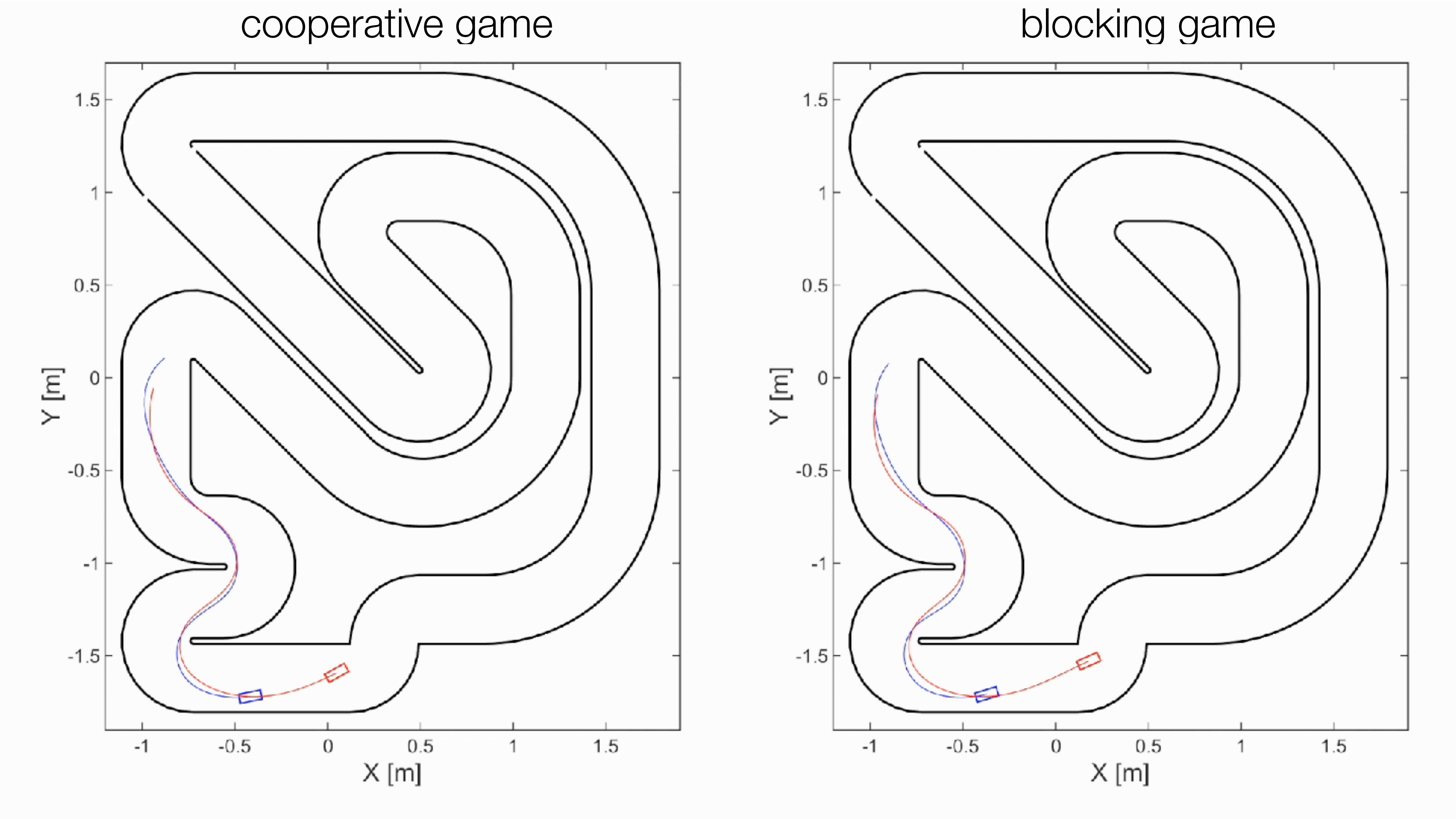
Simulation



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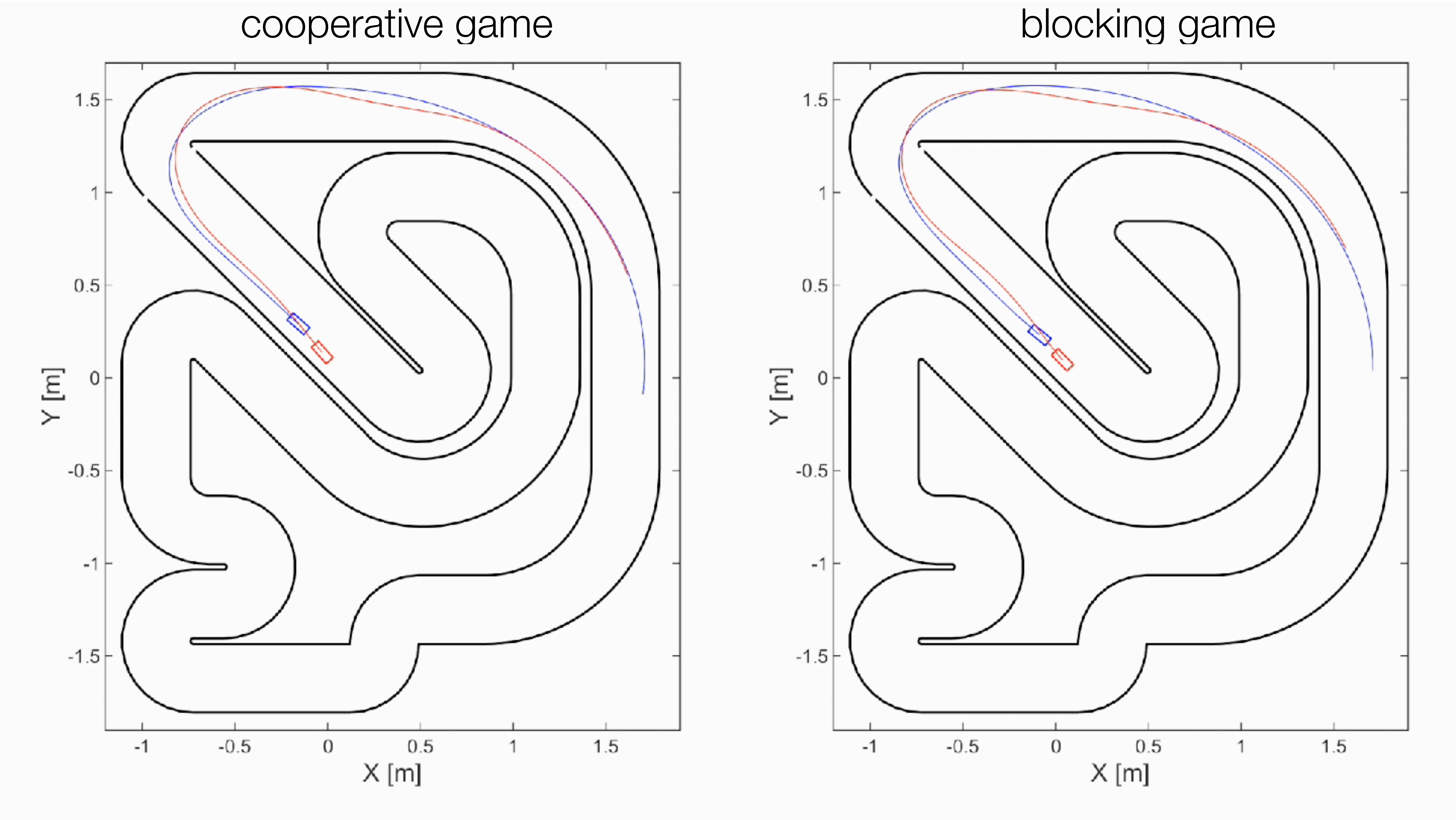

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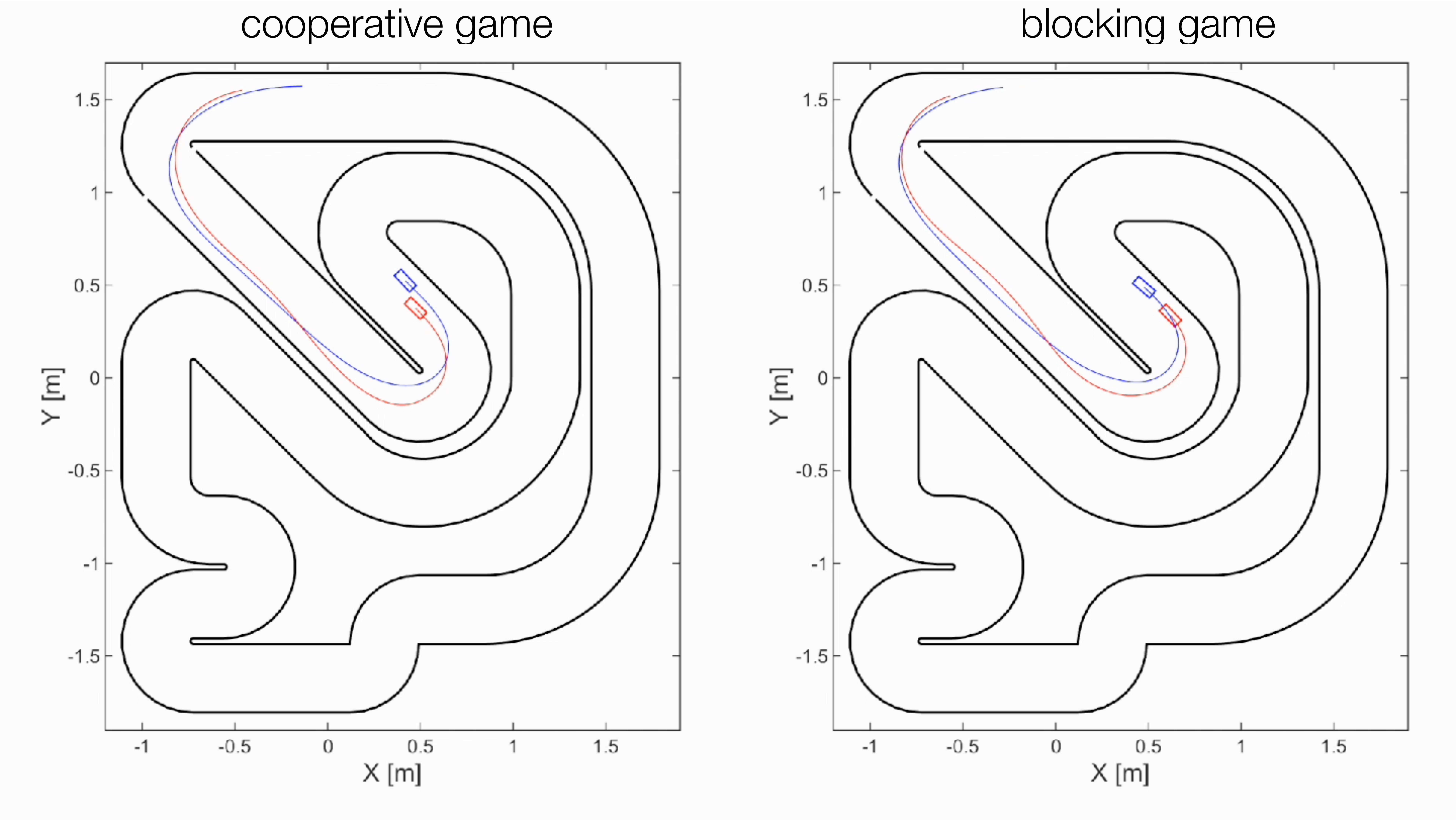

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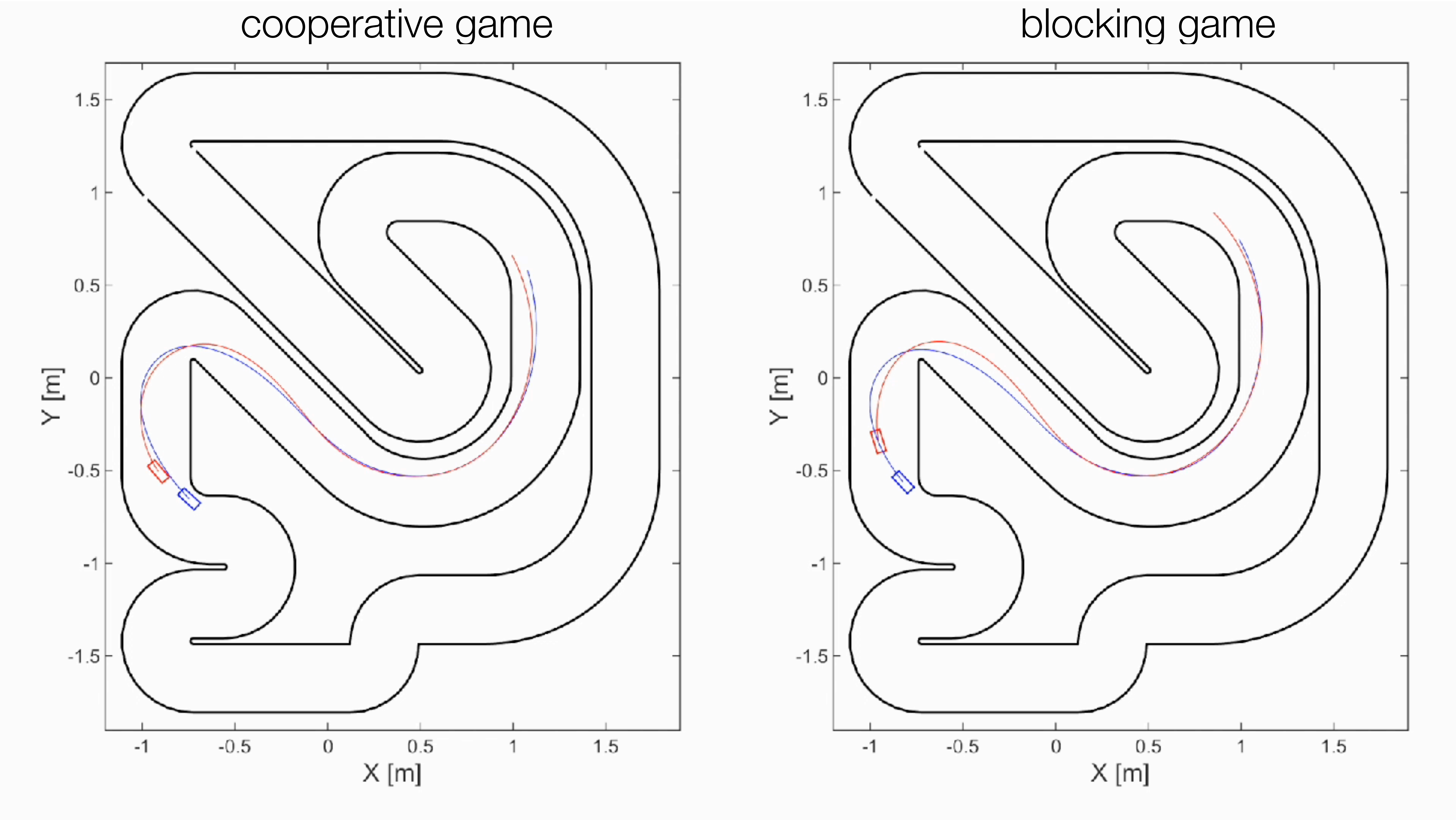

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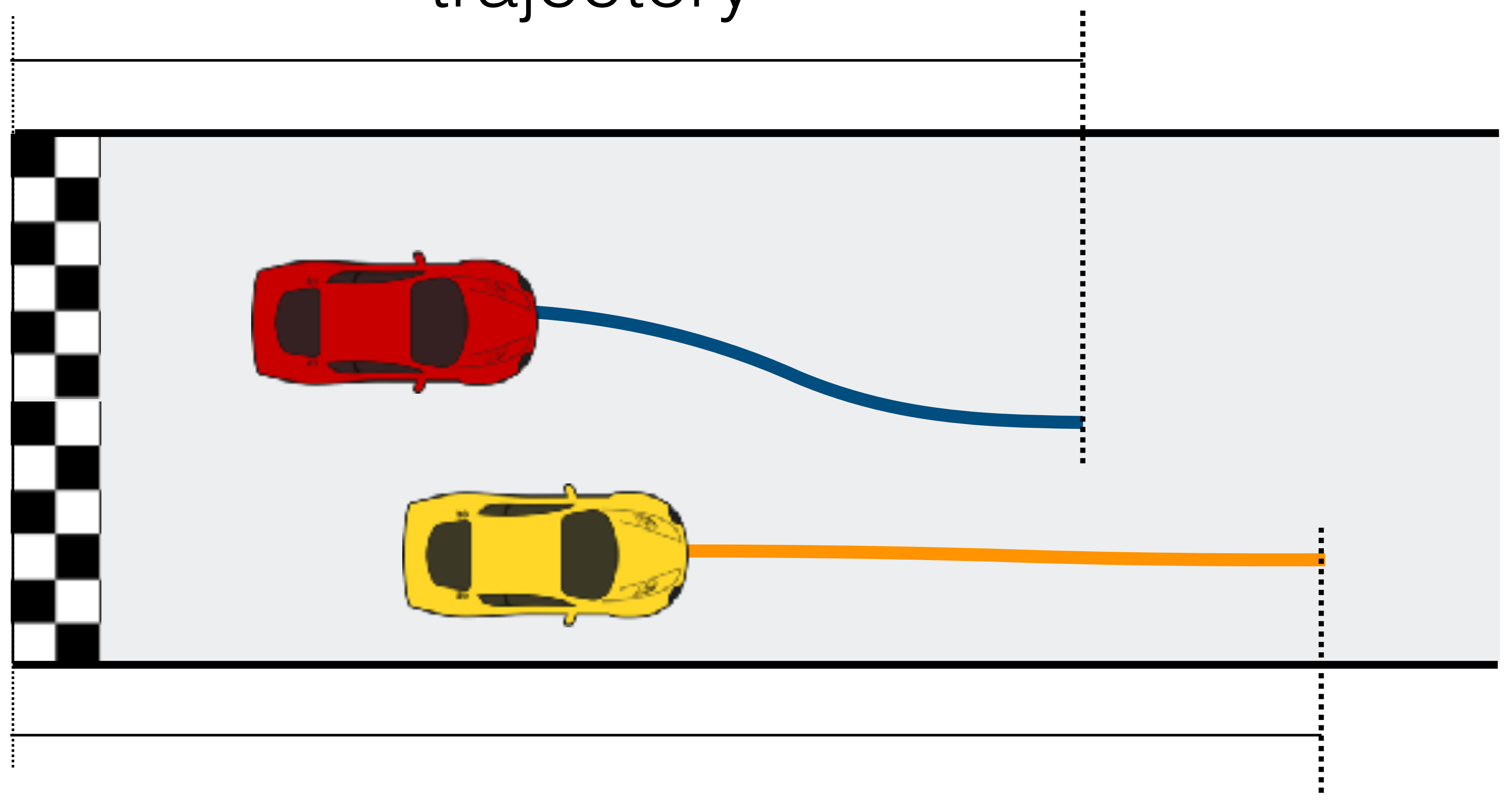
Real-Time Implementation

- Building the bimatrix game can be computationally expensive
  - Entries in the matrix grow quadratically with the number of trajectories
  - Collision checks become a bottle neck
  - 1,000 trajectories —> 32,000,000 collision checks (20ms discretization)
- Reduce prediction horizon from 3 to 2 steps (~200 instead of 3,000 trajectories)
- Only build bimatrix game for the 60 best trajectories (beam search)

Progress reward = progress of the trajectory



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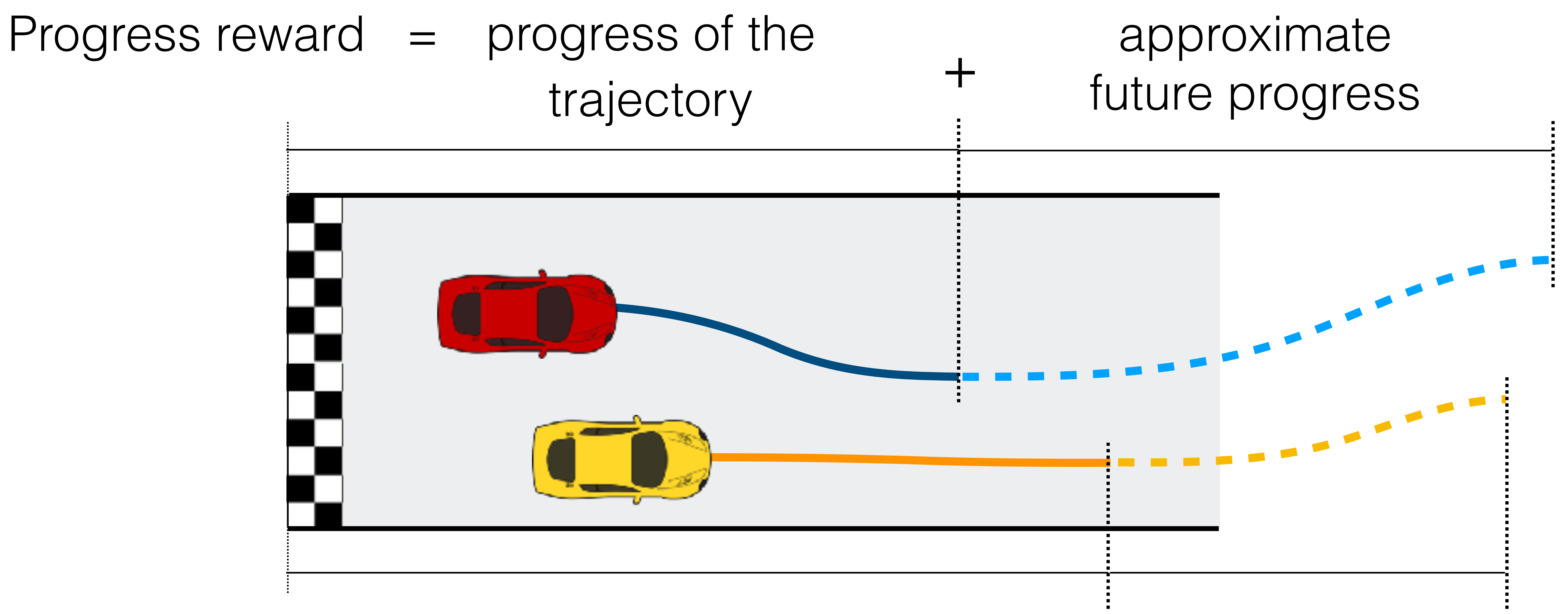

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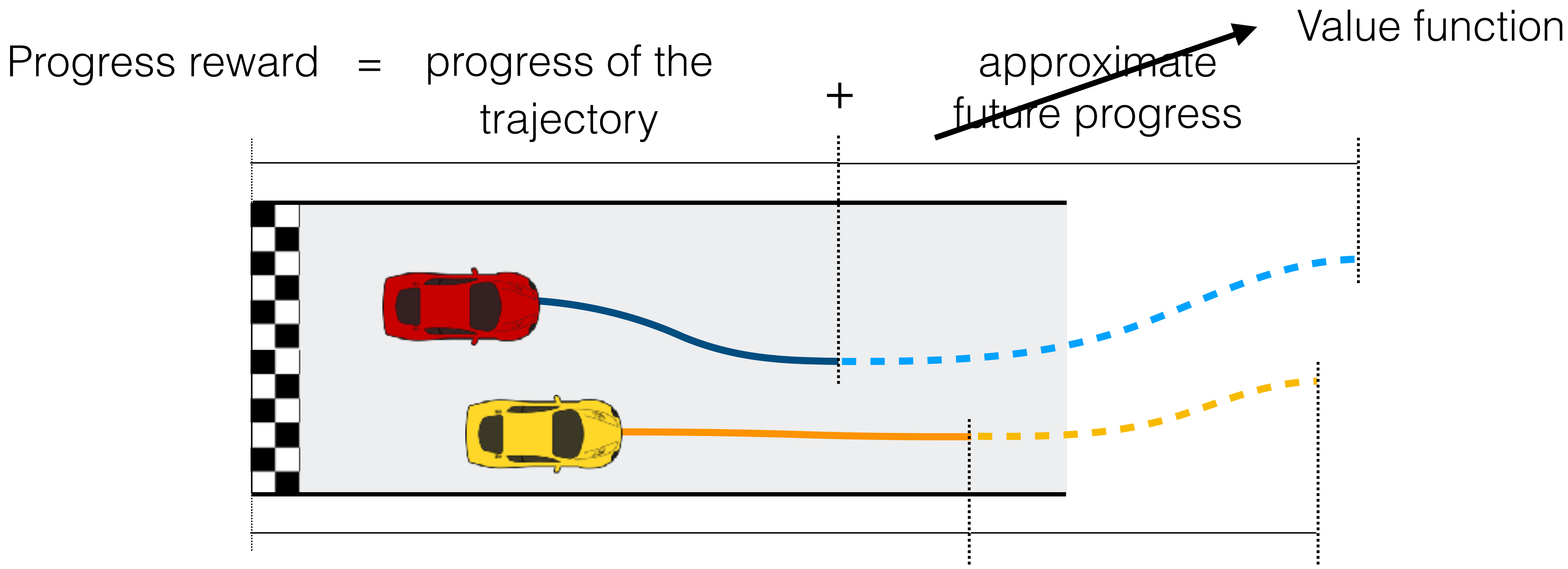

Experimental Results

Experimental Results

Summary and Outlook

- We proposed a complet autonomous racing pipeline
  - Different behavior is seen for different viability kernels and games
- Interesting insights into behavior of noncooperative decisions
  - Sequential maximization and leader-follower structure
- Reliable and real-time feasible games
  - Consider uncertainty in game formulation
  - Reinforcement learning-based terminal cost+constraints
  - High-performance implementation using GPU
- Model-learning for MPC

$$x_{k+1} = f(x_k, u_k) + \mu_{\text{GP}}(x_k, u_k)$$

- Learn behavior of “opponent” —> urban traffic

Questions