Autonomous Racing by Model Predictive Contouring Control

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Autonomous Driving

- DARPA Grand & Urban Challenge
- Google: >1.1 Mio km driven as of 4/14
- Tesla: >90% autonomous in 2016
- Mercedes: autonomous E & S-class
- BMW: autonomous drift with 235i
- ... and many others.

Autonomous Racing

- Autonomous Racing Audi TTS (Stanford University):

No online path planning / obstacle avoidance / overtaking

ORCA Autonomous Racing

Goal: Autonomous racing with obstacle avoidance

Outline

- Introduction
- Hardware Setup
- Car Modeling
- Model Predictive Contouring Control
- Obstacle Avoidance
- Solution Approach
- Results
- Current/Future Work

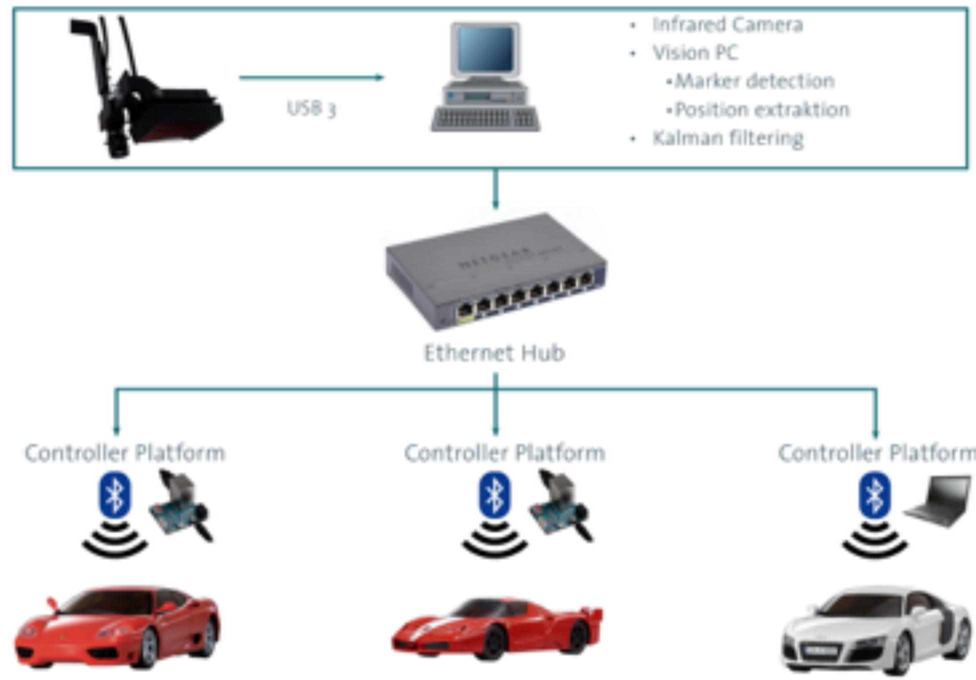
The ORCA Race Car Setup

- Built by ETH students from standard components using Kyosho dnano 1:43 RC cars

- Multiple control boards with RF connection to embedded car

- Embedded board inside the car:
  - ARM M4 microcontroller
  - Gyro and accelerometer
  - H-bridges for motors
  - Voltage & current measurement
  - Comms (Bluetooth)

http://control.ee.ethz.ch/~racing/



Camera System

- Infrared based vision system
- PointGrey Flea3
  - 100 fps
  - resolution ~3.5mm

- 7 unique marker patterns

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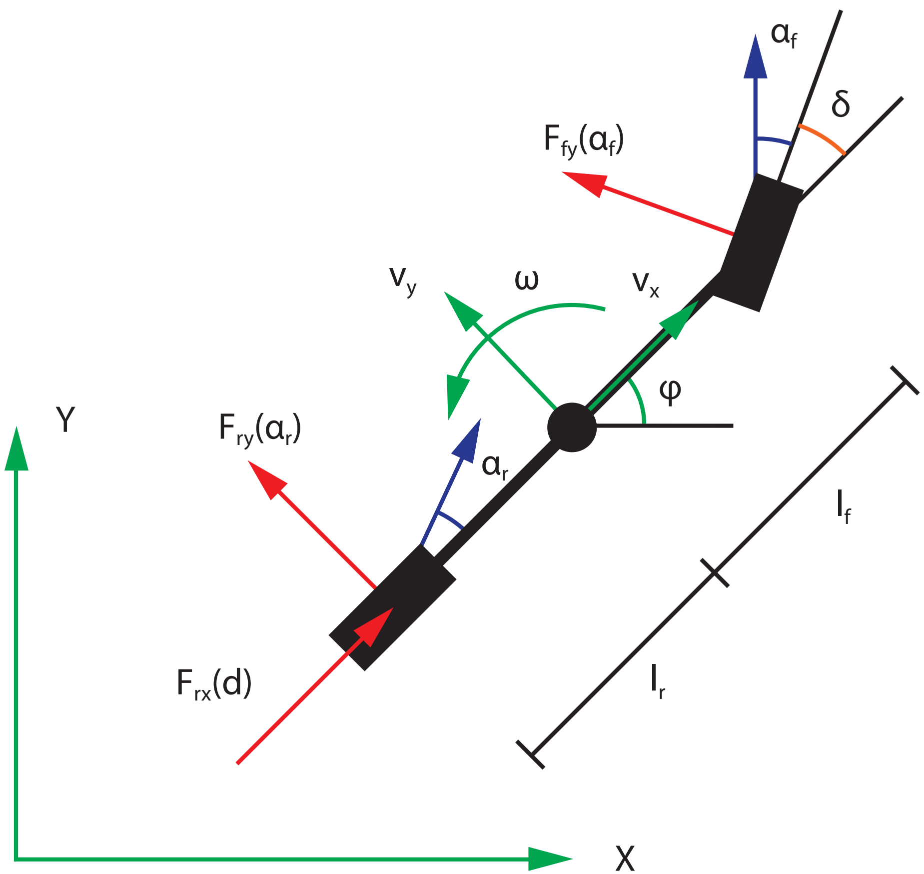
Car Model

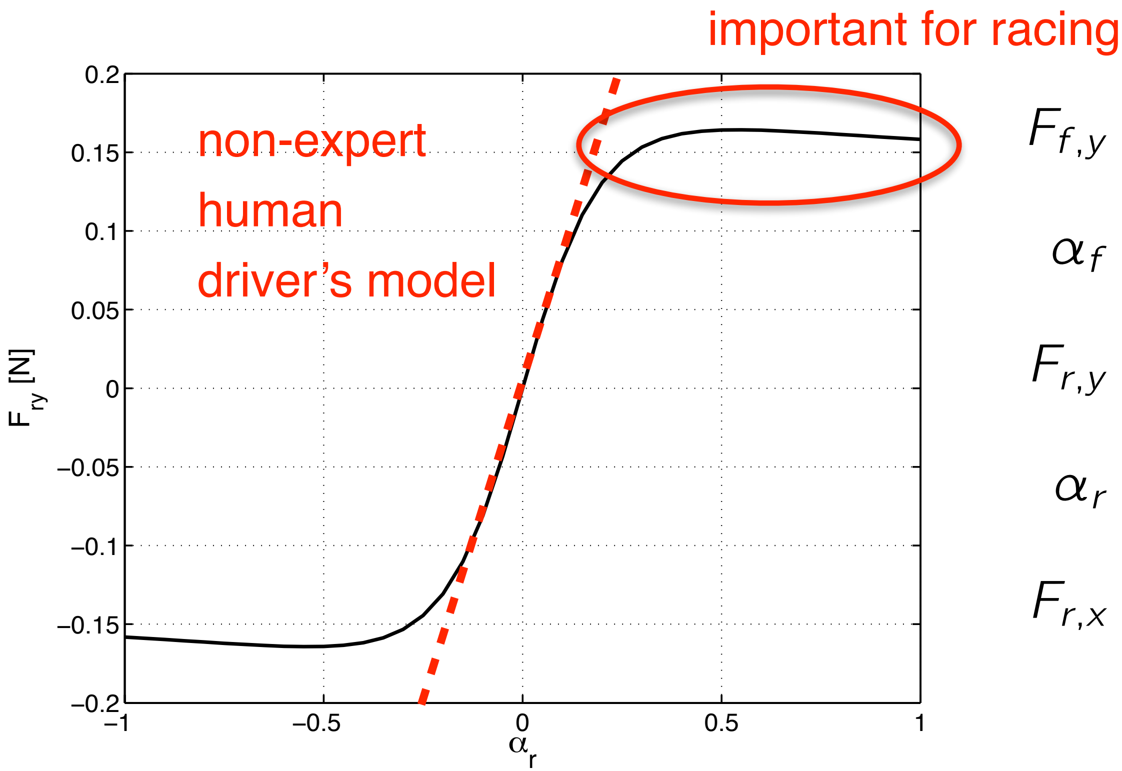
- Bicycle model with generic tire forces



$$\dot{X} = v_x \cos(\varphi) - v_y \sin(\varphi)$$
$$\dot{Y} = v_x \sin(\varphi) + v_y \cos(\varphi)$$
$$\dot{\varphi} = \omega$$
$$\dot{v}_x = \frac{1}{m}(F_{r,x} - F_{f,y} \sin\delta + m v_y \omega)$$
$$\dot{v}_y = \frac{1}{m}(F_{r,y} + F_{f,y} \cos\delta - m v_x \omega)$$
$$\dot{\omega} = \frac{1}{I_z}(F_{f,y} l_f \cos\delta - F_{r,y} l_r)$$

Tire Force Model

- Pacejka's "magic formula"



$$F_{f,y} = D_f \sin(C_f \arctan(B_f \alpha_f))$$

$$\alpha_f = -\arctan\left(\frac{\omega l_f + v_y}{v_x}\right) + \delta$$

$$F_{r,y} = D_r \sin(C_r \arctan(B_r \alpha_r))$$

$$\alpha_r = \arctan\left(\frac{\omega l_r - v_y}{v_x}\right)$$

$$F_{r,x} = (C_{m1} - C_{m2} v_x) d - C_r - C_d v_x^2$$

- Car inputs: duty cycle for DC motor d, steering angle δ
- Coefficients identified in various experiments

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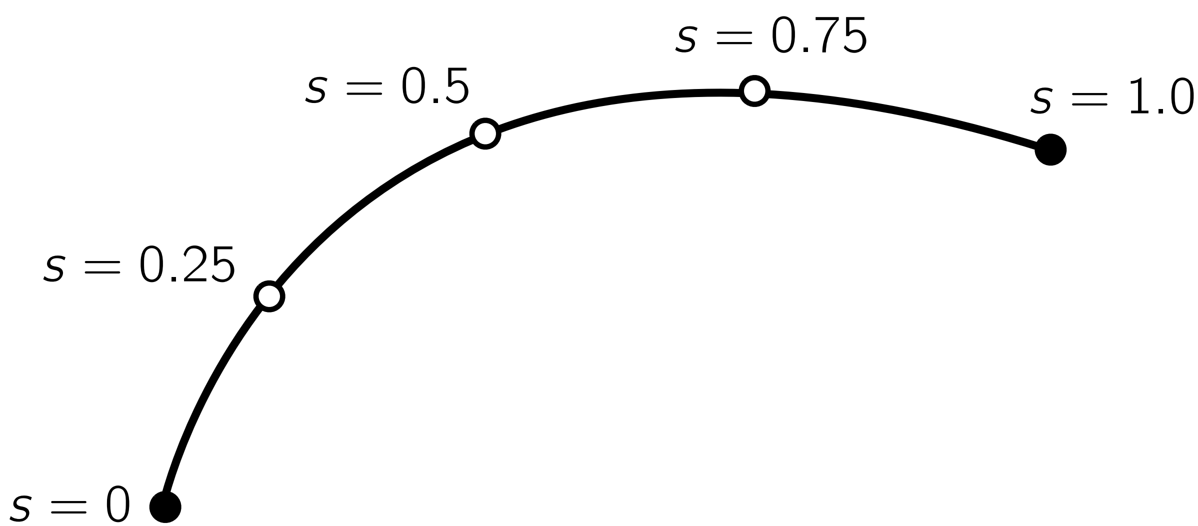
Path Parameterization

- Introduce variable $s \in [0, 1]$ denoting the normalized arc length of path:



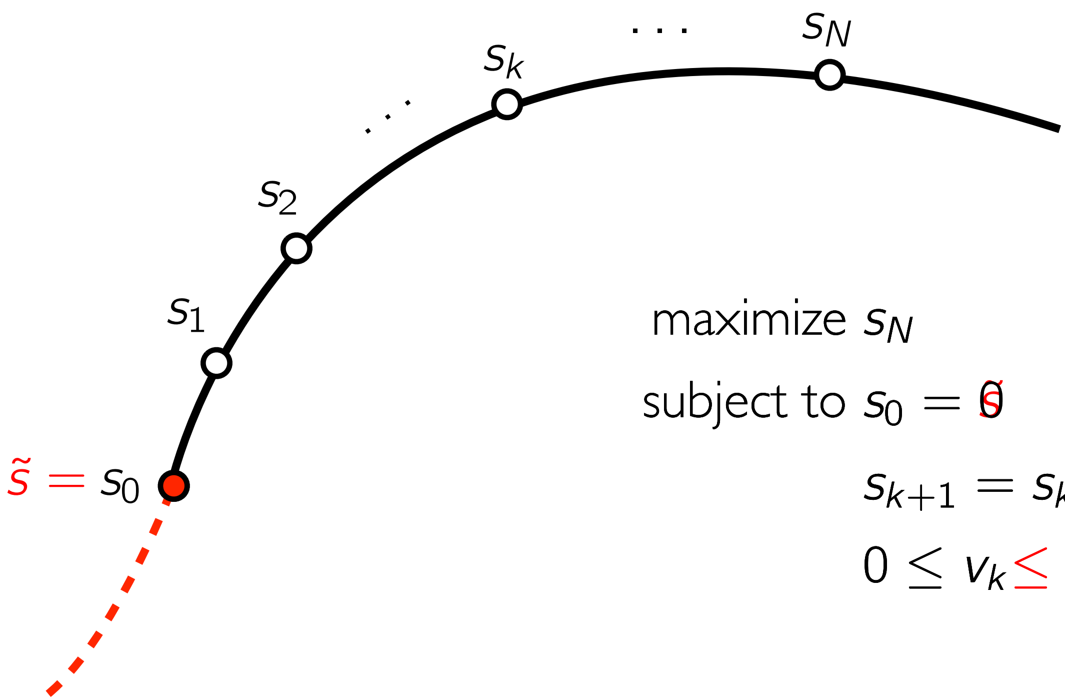
- Write discrete-time dynamics for arc length:

$$s_{k+1} = s_k + v_k \ , \quad v_k \geq 0$$

- v_k is (constrained) input

Minimum Time Objective

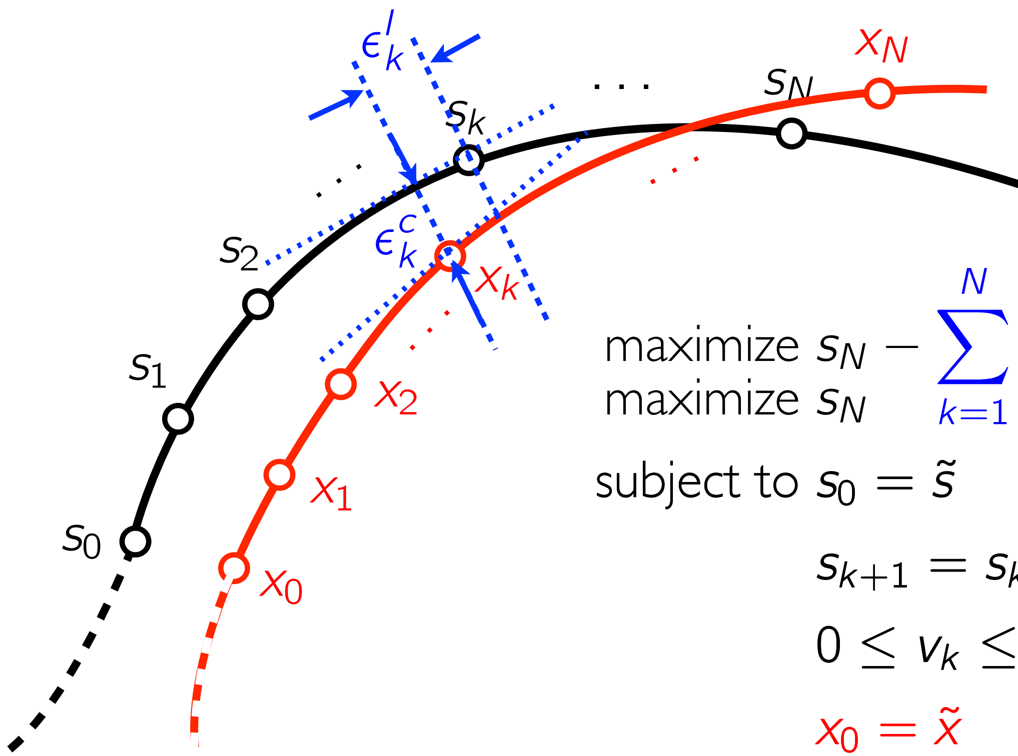
- maximize travelled arc in N time steps $\Leftrightarrow$ minimize time to travel s_N (assuming $s_N < 1$)



$$
\begin{aligned}
&\text{maximize } s_N \\
&\text{subject to } s_0 = \cancel{0}\,\tilde{s} \\
&\qquad s_{k+1} = s_k + v_k\,,\ k = 0, \dots, N-1 \\
&\qquad 0 \leq v_k \leq \bar{v}
\end{aligned}
$$

Linking the Physical System via Path Errors

- Adding physical dynamics:



$$\text{maximize } s_N - \sum_{k=1}^{N} \gamma_c \|\epsilon_k^c\|^2 + \gamma_l \|\epsilon_k^l\|^2$$

$$\text{maximize } s_N$$

$$\text{subject to } s_0 = \tilde{s}$$

$$s_{k+1} = s_k + v_k \,, k = 0, \ldots, N-1$$

$$0 \le v_k \le \bar{v}$$

$$x_0 = \tilde{x}$$

$$x_{k+1} = f(x_k, u_k) \,,\ k = 0, \ldots, N-1$$

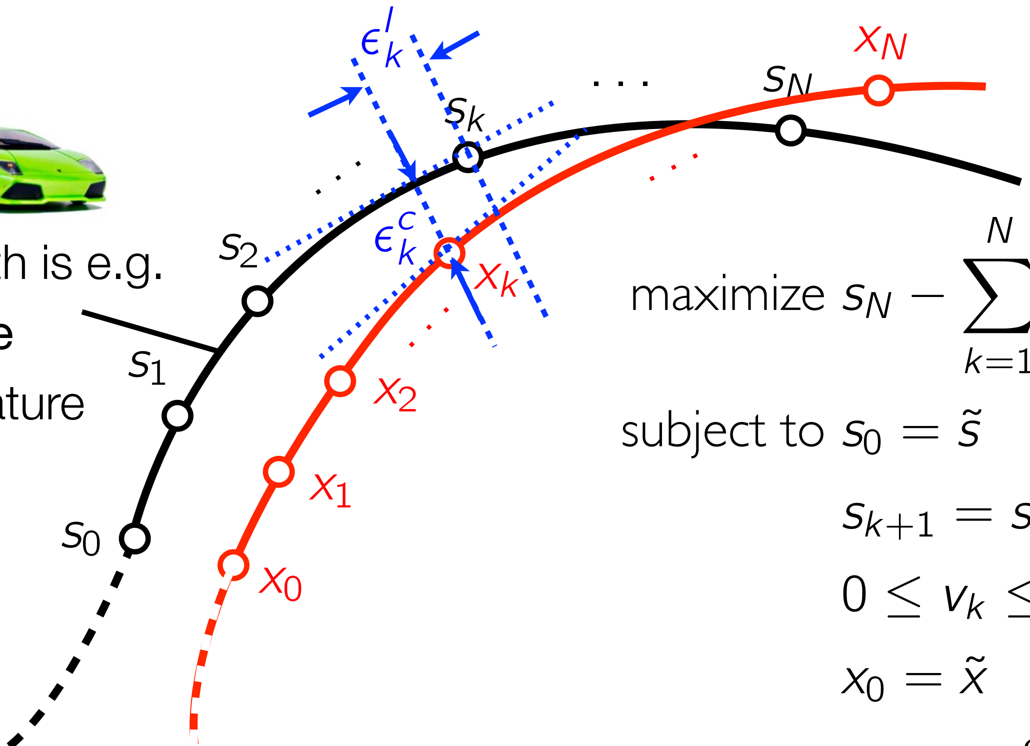
$$x_k \in \mathbb{X}_k, u_k \in \mathbb{U}_k$$

$$\epsilon_k^c = g(x_k, s_k) \,,\ \epsilon_k^l = h(x_k, s_k)$$

- Minimization of contouring errors ϵ_k^c and lag errors ϵ_k^l couples car dynamics to minimum time problem

Contouring Control Adapted for Racing

- **Machine tools:** want to follow a given path accurately
  - Contouring error ϵ_k^c must be low - γ_c must be high



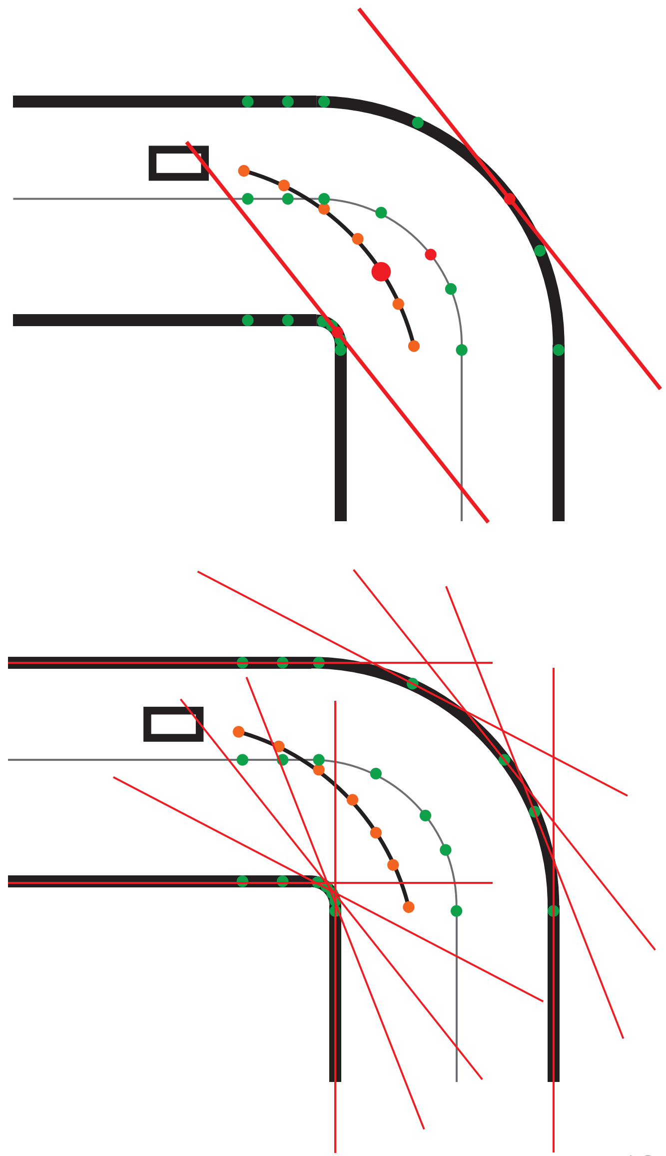
Racing: path is e.g.
- **center line**
- min. curvature path

$$
\begin{aligned}
\text{maximize } & s_N - \sum_{k=1}^{N} \gamma_c \|\epsilon_k^c\|^2 + \gamma_l \|\epsilon_k^l\|^2 \\
\text{subject to } & s_0 = \tilde{s} \\
& s_{k+1} = s_k + v_k \,,\ k = 0, \ldots, N-1 \\
& 0 \leq v_k \leq \bar{v} \\
& x_0 = \tilde{x} \\
& x_{k+1} = f(x_k, u_k)\,,\ \ k = 0, \ldots, N-1 \\
& x_k \in \mathbb{X}_k,\, u_k \in \mathbb{U}_k \\
& \epsilon_k^c = g(x_k, s_k)\,,\ \epsilon_k^l = h(x_k, s_k)
\end{aligned}
$$

- **Racing:** lateral deviation from path is OK - choose γ_c low

Race Track Constraints

- Track constraints in general non-convex
- Each point in the horizon is constrained within two half spaces (state constraints)
- Points of previous prediction are used to generate constraints (projection on track borders)



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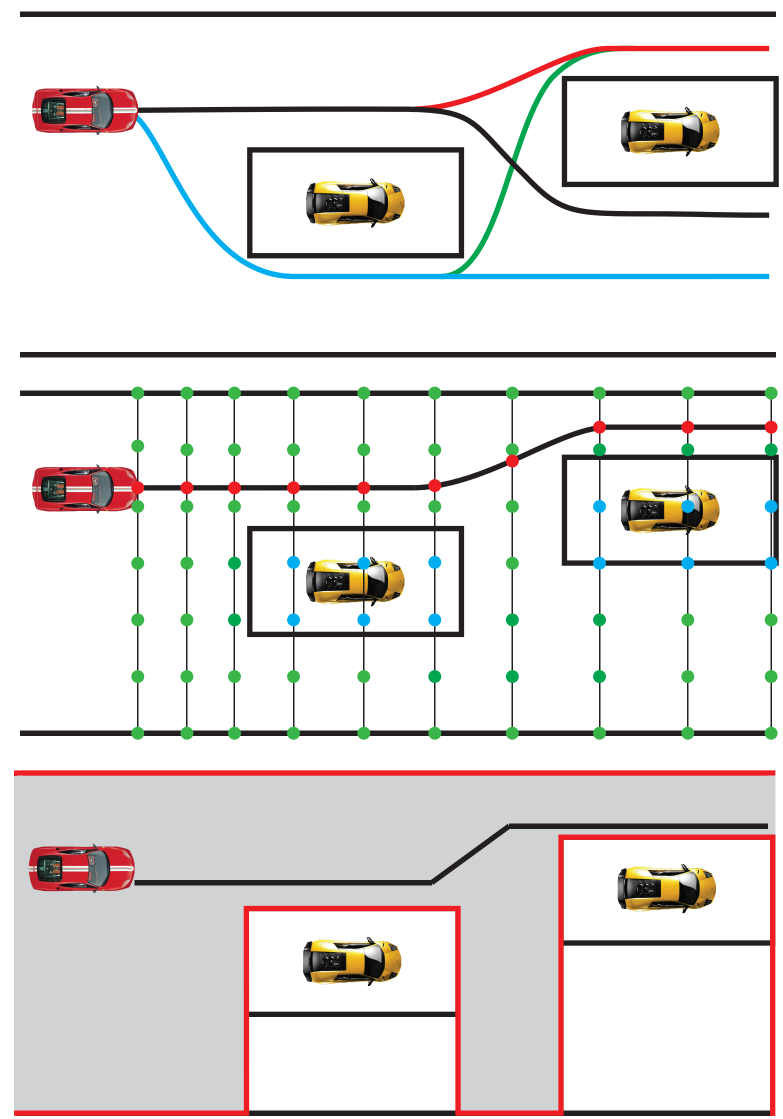
Obstacle Avoidance via Track Morphing

- Overtaking problem is non-convex
- 2-level solution approach:
  - high-level controller computes feasible "corridor" (convex set)
  - MPCC uses this corridor
- Dynamic programming computes optimal corridor avoiding opponents
  - Generate a spatial-temporal grid, based on the last MPCC iteration
  - minimize travelled distance and deviation from the last trajectory
- Morph feasible set for MPCC based on optimal corridor



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Nonlinear MPC Implementation

- Real-time iteration/multiple shooting algorithm [Diehl 2002] with matrix exponential as integrator. In each time step,

1. Get current position $\tilde{x}$
2. Construct new state trajectory with $\tilde{x}$ and new $x_N \equiv x_{N-1}$
3. Generate new track constraints
4. **Linearize** continuous-time dynamics around trajectory
5. **Discretize** using matrix exponential
6. Solve **local convex approximation** (QP) $\longrightarrow$
7. Apply first input

$$
\begin{aligned}
\max\ & s_N - \sum_{k=1}^{N} \gamma_c \|\epsilon_k^c\|^2 + \gamma_l \|\epsilon_k^l\|^2 \\
\text{s.t. } & s_0 = \tilde{s}\,, \ \ x_0 = \tilde{x} \\
& s_{k+1} = s_k + v_k \\
& 0 \leq v_k \leq \bar{v}\,, \ \ x_k \in \mathbb{X}_k, u_k \in \mathbb{U}_k \\
& x_{k+1} = A_k x_k + B_k u_k + g_k \\
& \epsilon_k^c = E_k x_k + F_k s_k + f_k \\
& \epsilon_k^l = G_k x_k + H_k s_k + h_k
\end{aligned}
$$

- Can be shown to converge to local optimum
- Initialization: run iterations from above until convergence

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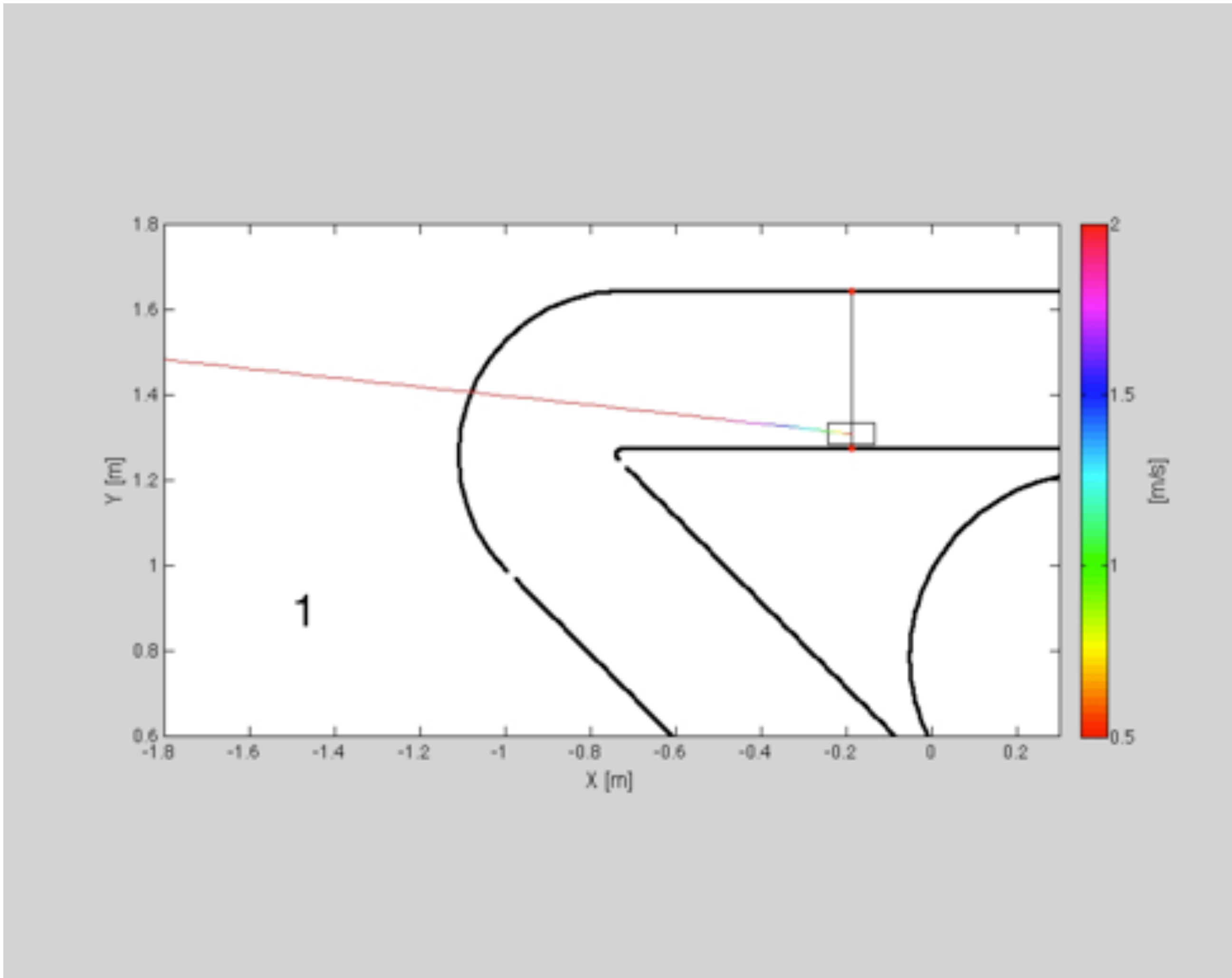
Computation Times

- Computation times in milliseconds on Exynos 5410 (ARM Cortex A15 @1.6 GHz)

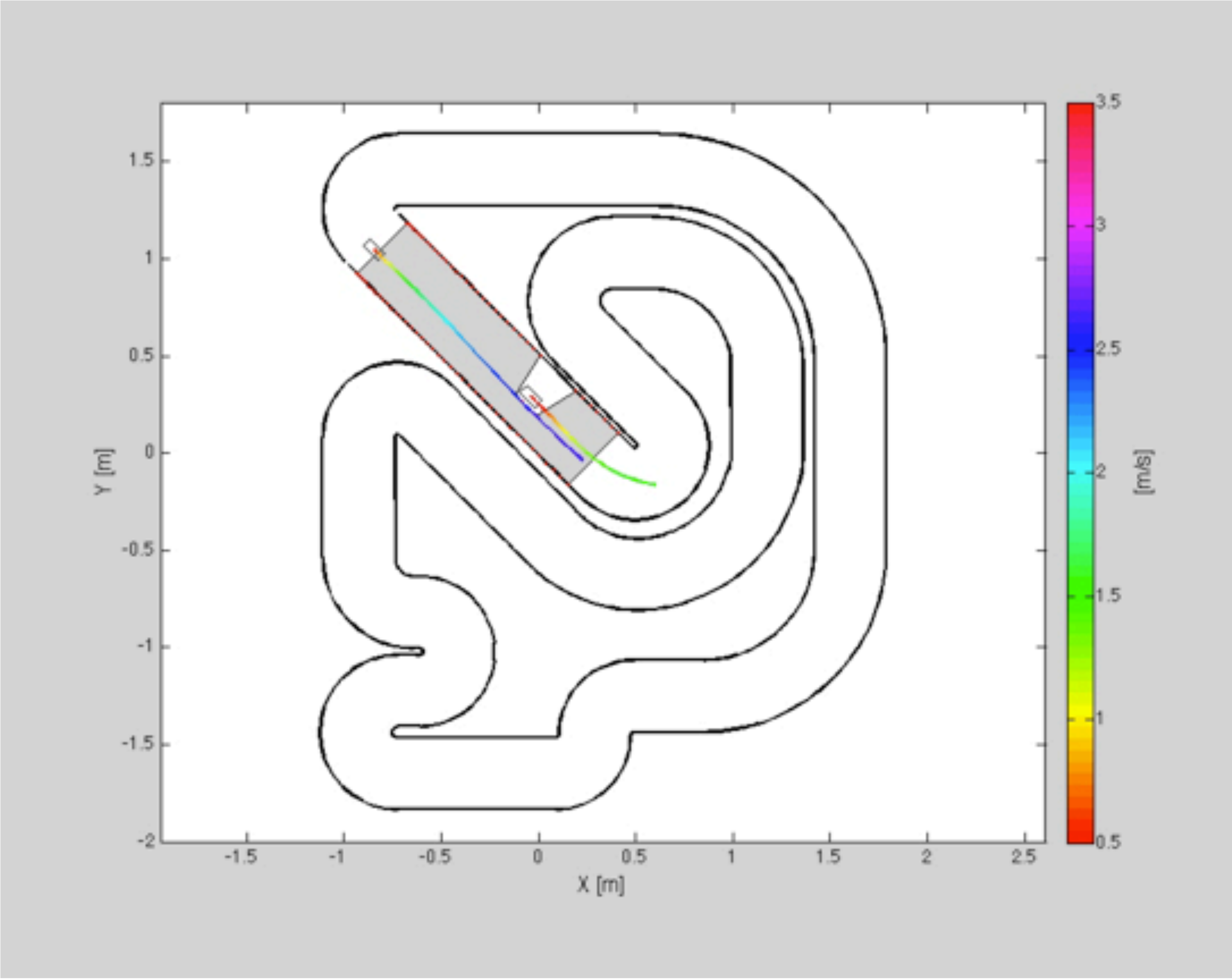
	Mean	Stdev	Max
Border Adjustment	0.7	0.24	1.05
QP Generation	2.34	0.37	2.79
QP with FORCES	14.43	1.40	21.46

- Sampling rate of 50 Hz possible (4.4% overtime)
- Problem dimensions:
  - Variables per stage: 13
  - Constraints per stage: 16
  - Horizon length: 40
- Total: QP with 520 variables, 640 constraints, LTV dynamics

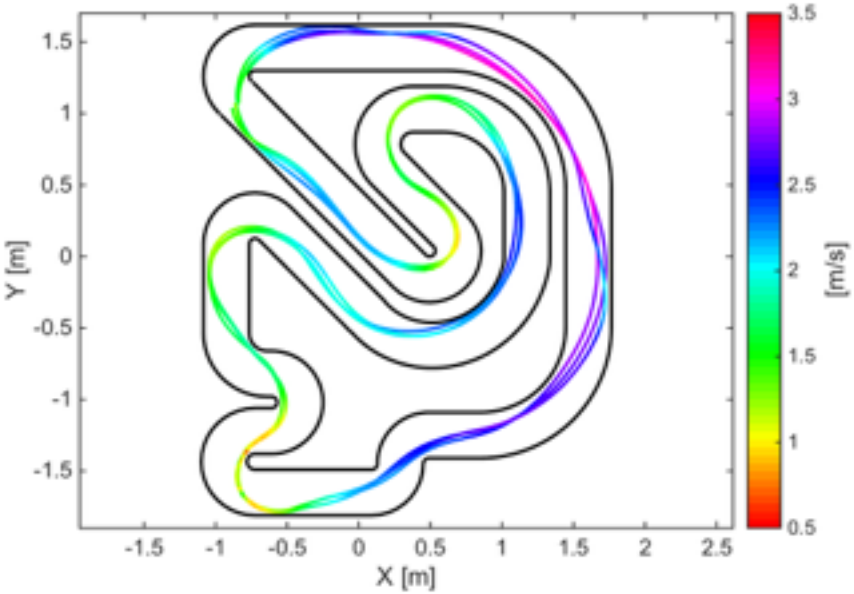
Initialization

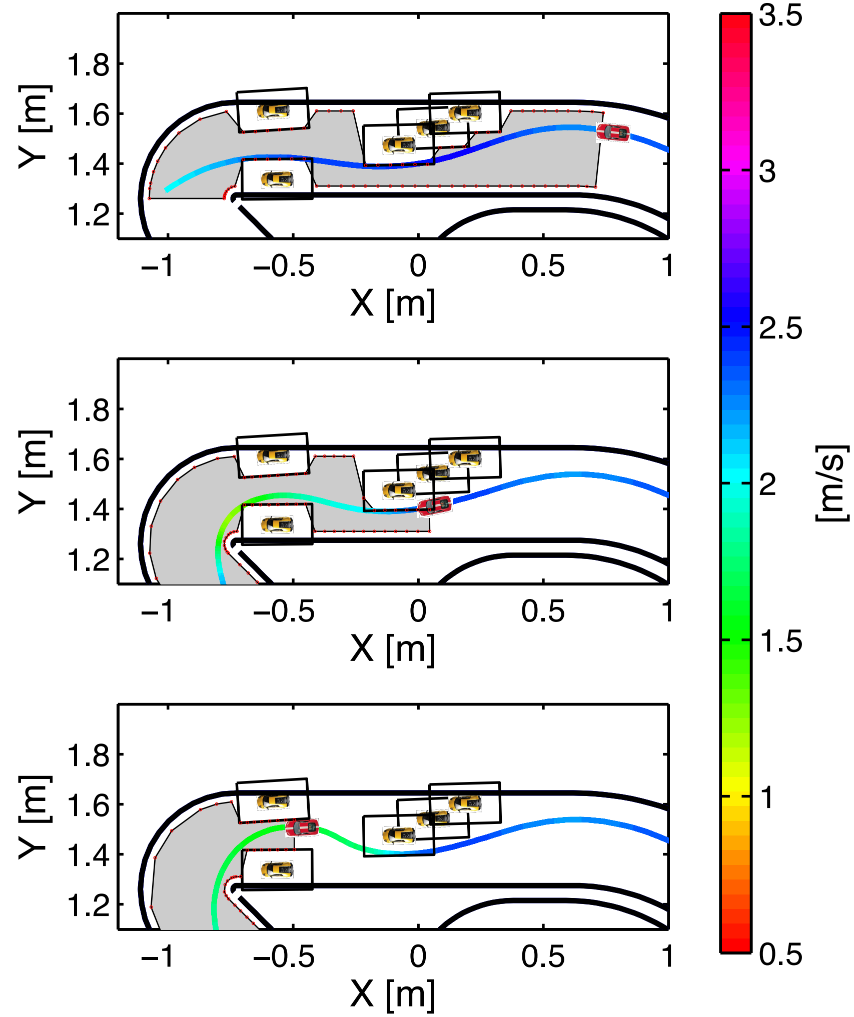


Closed-Loop Simulation

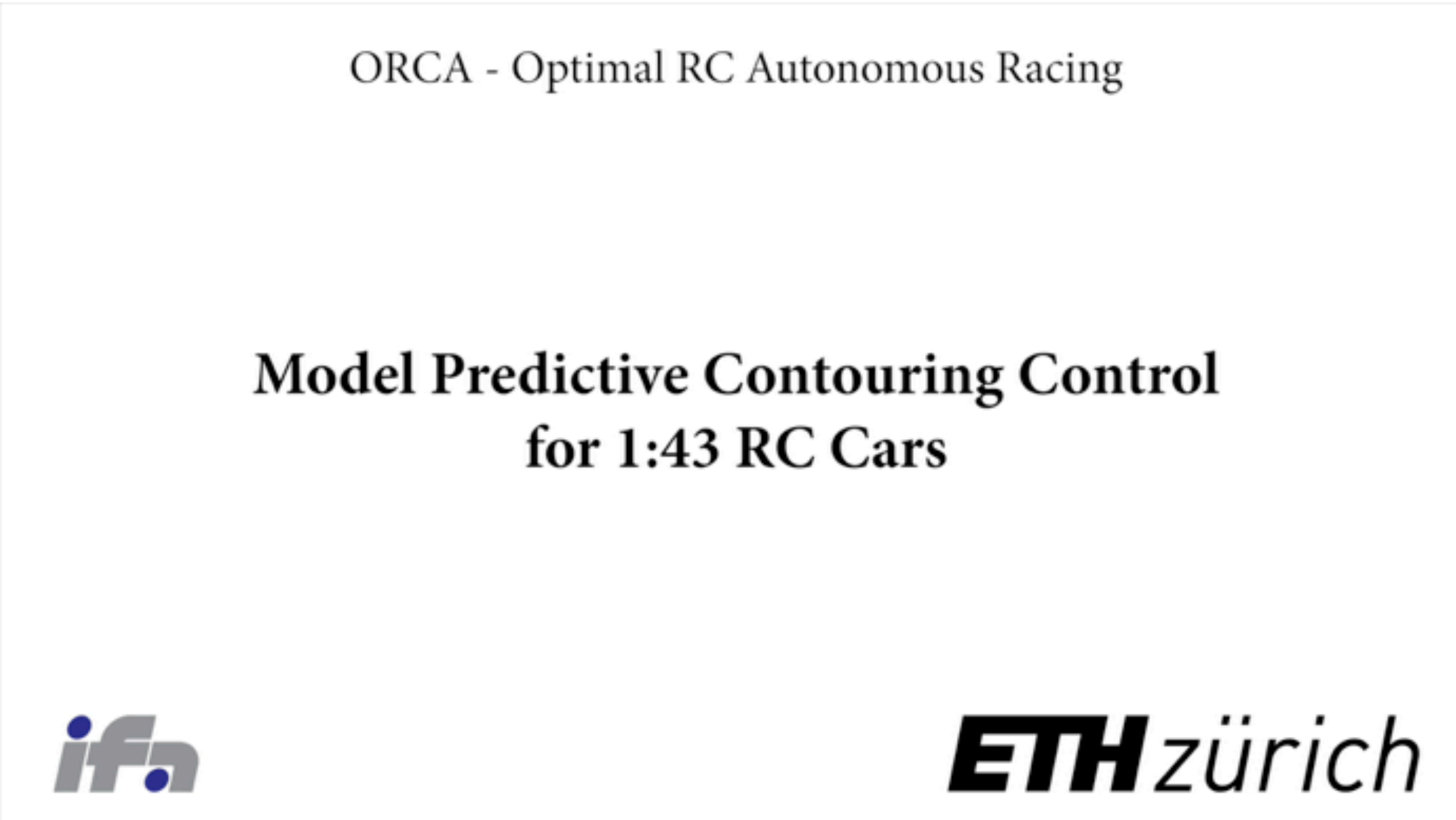


Closed Loop Results



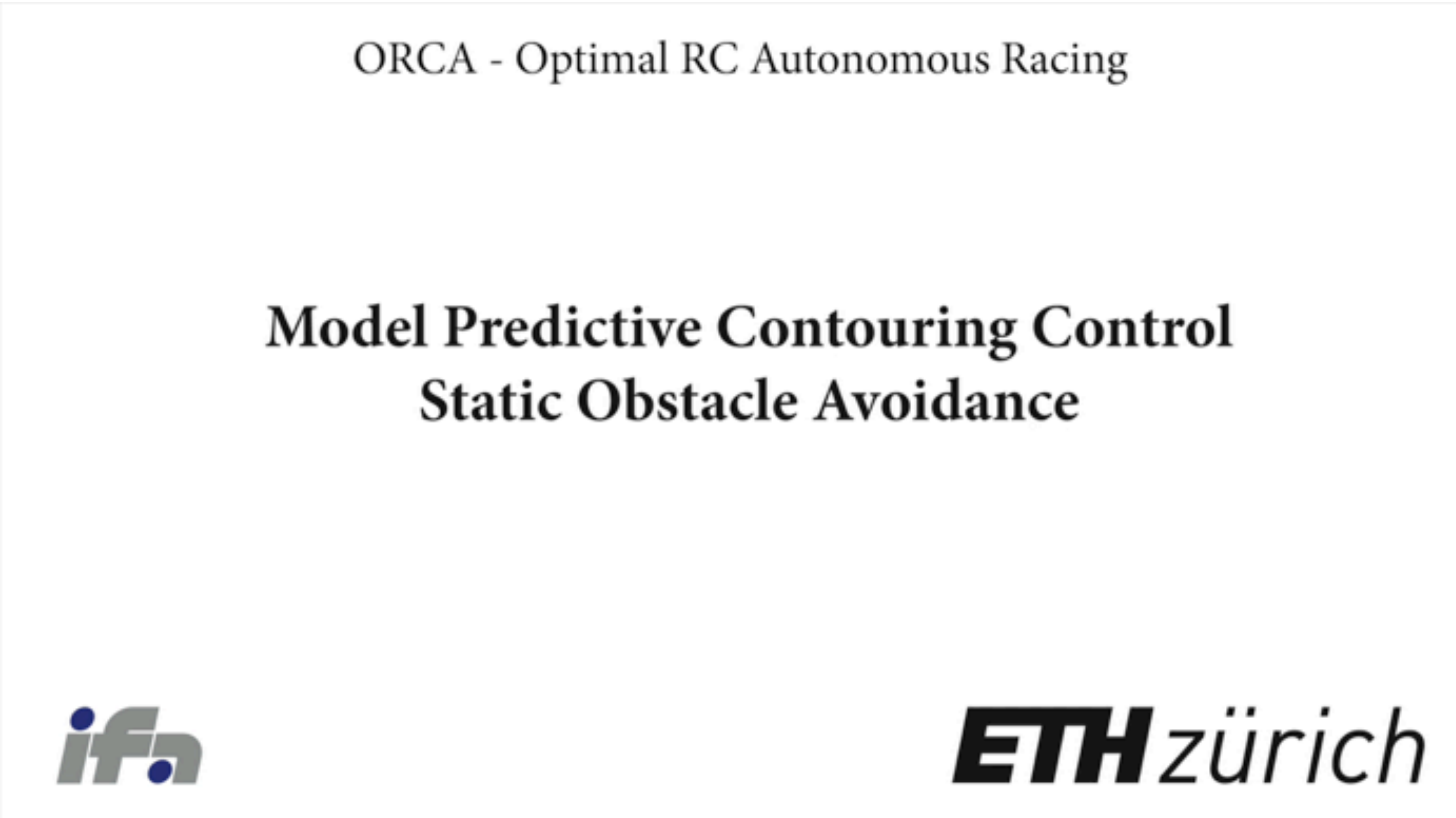


Autonomous RC Racing Using MPCC



https://www.youtube.com/watch?v=w46kkjKda9s

Obstacle Avoidance



https://www.youtube.com/watch?v=JoHfJ6LEKVo

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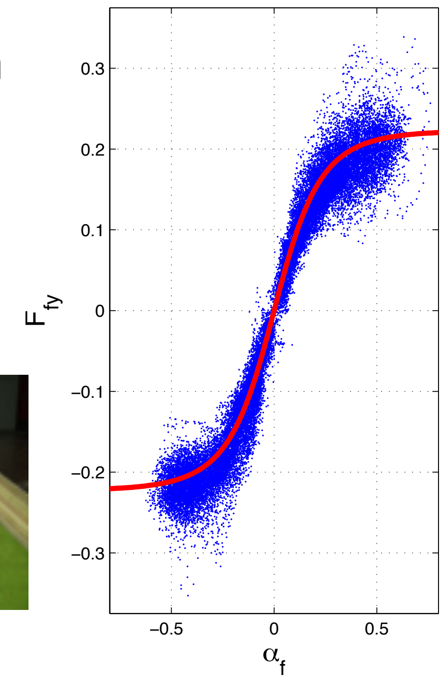
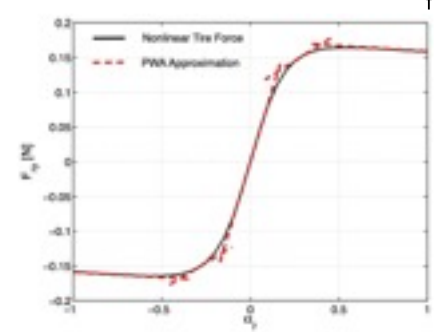
Additional Work using MPCC

- Scenario MPC
  - Additive error describing “any” kind of model mismatch
  - difference between prediction and measured state
  - “model-based” constraint tightening
  - open-loop control inputs are too conservative



- Driving Backwards
  - same model for driving backwards
  - a lot harder than expected

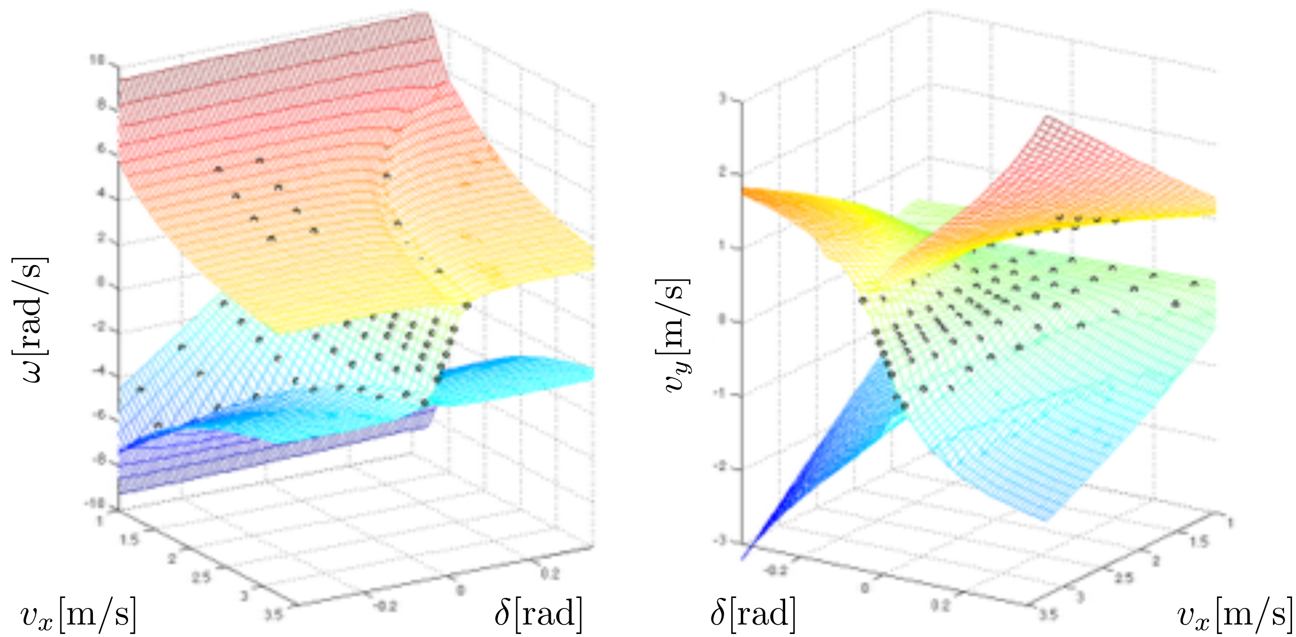
- Piecewise-affine tire forces (on going)
  - goal: using inverse optimization to solve problem efficiently



Hierarchical Control

1. **Path planning** based on motion primitives
2. **MPC** tracking optimal trajectory

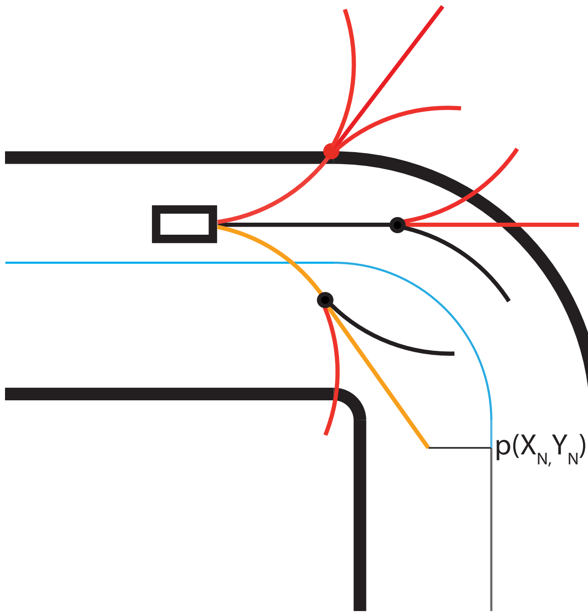
- Motion primitives / Constant velocity points:



- Directly linking trims if the transition is feasible for bicycle model

Hierarchical Control

- Splits at fixed sampling times



- Assumption:
  - new constant velocity can be reached immediately
- Model simplifies

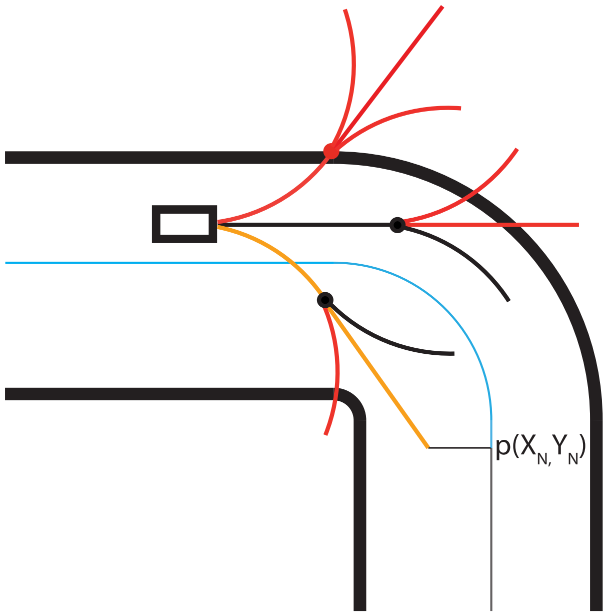
$$\dot{X} = \bar{v}_x(q)\cos(\varphi) - \bar{v}_y(q)\sin(\varphi)$$
$$\dot{Y} = \bar{v}_x(q)\sin(\varphi) + \bar{v}_y(q)\cos(\varphi)$$
$$\dot{\varphi} = \bar{\omega}(q)$$
$$\bar{v}(q) = [\bar{v}_x(q), \bar{v}_y(q), \bar{\omega}(q)]$$

$$p(X,Y) := \arg\min_{\theta}(X - X^{\text{cen}}(\theta))^2 + (Y - Y^{\text{cen}}(\theta))^2$$

- Can be modeled as hybrid system and analyzed formally

Hierarchical Control

- Splits at fixed sampling times



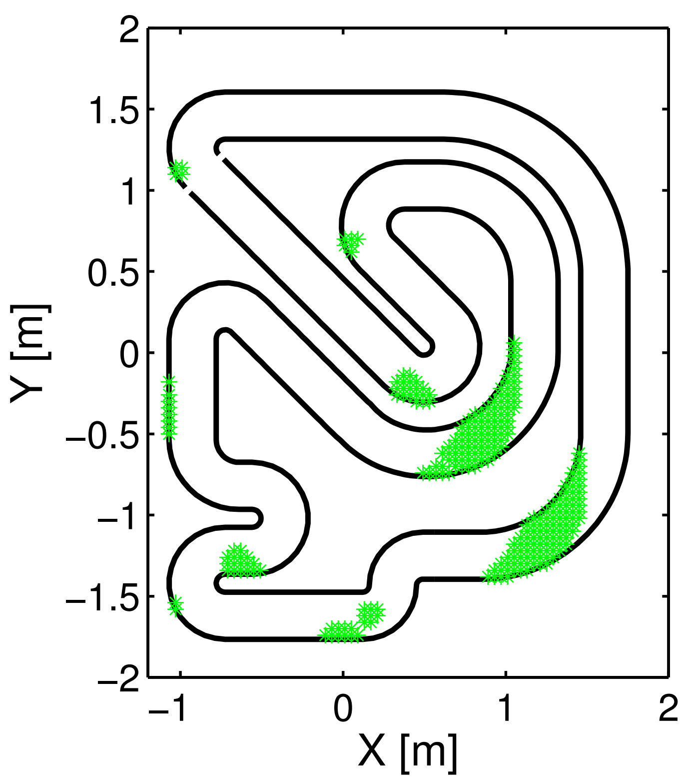
- Assumption:
  - new constant velocity can be reached immediately
- Find trajectory:
  - with maximal progress
  - which does not leave the track
- Tree grows exponentially

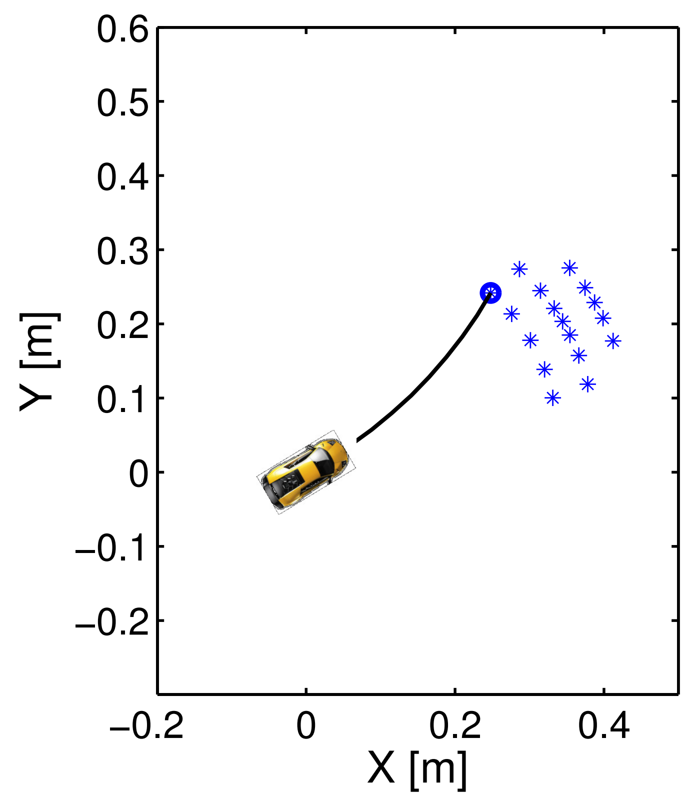
$$p(X,Y) := \arg\min_{\theta}(X - X^{\text{cen}}(\theta))^2 + (Y - Y^{\text{cen}}(\theta))^2$$

- Can be modeled as hybrid system and analyzed formally

Fast Viable Path Planning

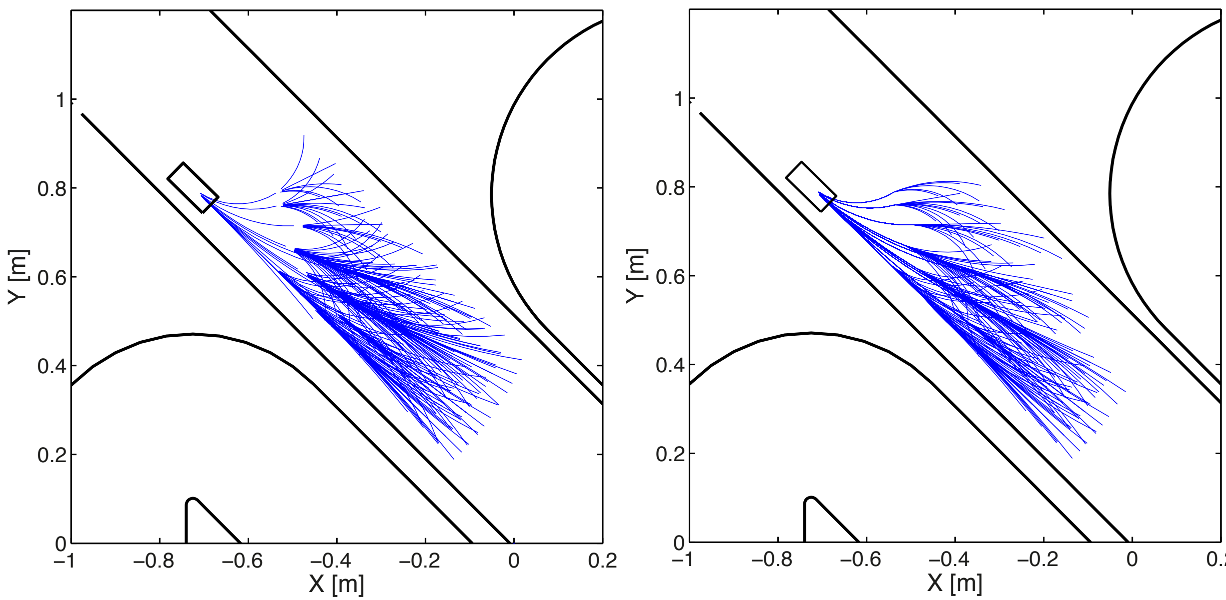
- Based on viability theory (invariant set theory)
  - reconstructing all “safe” motion primitives given the state
- Driving straight with 2m/s with a fix angle





Fast Viable Path Planning

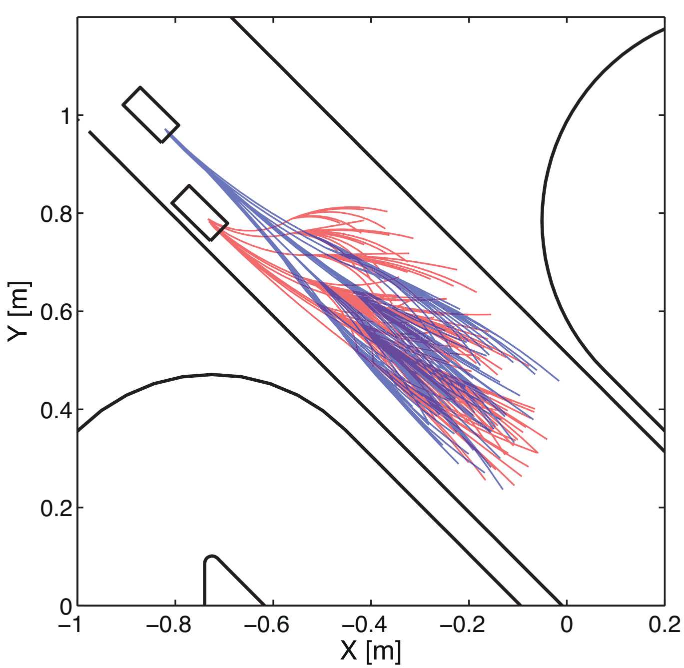
- Only generating trajectories which are recursive feasible
  - only generate 145 vs 341 trajectories
  - no constraint checks necessary

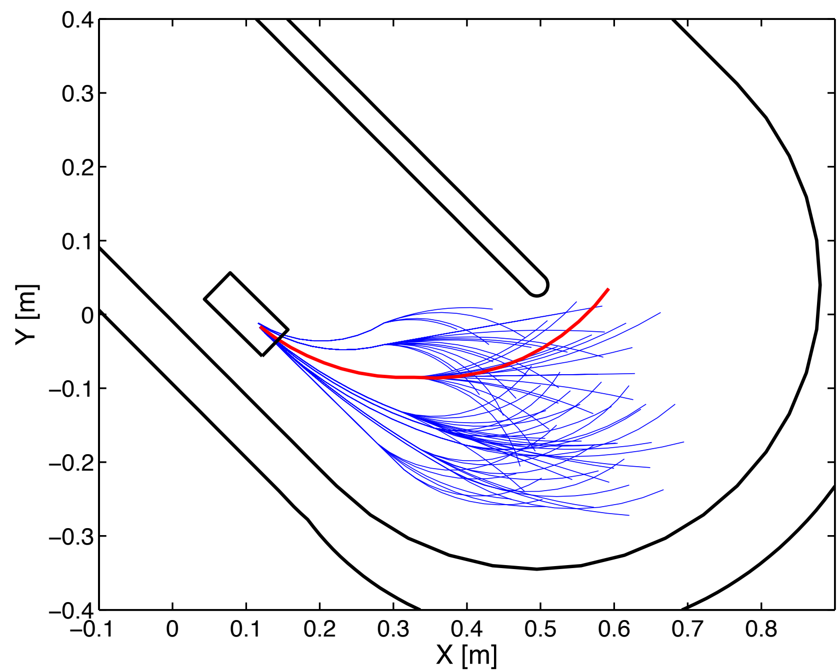


	Naive Path Planner	$\mathcal{V}_{\text{safe}}(x)$ Path Planner
mean [s]	0.219	0.0205
max [s]	0.530	0.0749
min [s]	0.034	0.003

Prediction of Opponent

- Efficient and fast calculation of possible movements
- Could be used for dynamic obstacle avoidance
  - robust or probabilistic collision avoidance
  - store one optimal trajectory (we showed that this is a Nash equilibrium)





Acknowledgments

- Students involved in MPCC:
  - Florian Perrodin (First MPCC Simulations)
  - Samuel Zhao (MPCC Implementation I)
  - Kenneth Kuchera (MPCC Implementation II)
  - Michael Janser (Dynamic Programming for obstacle avoidance)
  - Sandro Merkli (Embedded controllers & electronics)
  - Marcin Dymczyk (Embedded software stack & electronics)
  - Nils Wenzler (Control loop and communication framework)
  - Christian Stocker (Embedded hardware design)
  - Michael Dahinden (Embedded hardware design)
  - Simon Tanner (Vision System)

- Prof. Colin Jones & Prof. Manfred Morari

Closed Loop Results

- Lap time **8.5 - 9** s